Application of Metrical Matrix and set Notation to Rock Formining Minerals using Combined Goldschmidt and Bowen’s Concept with Respect to Thermodynamic Principles

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Abstract

Using mathematical context such as Metrical matrix and Set notation methods, the formation of rock forming minerals from magma, depends on the numerical value of each mineral under thermodynamic conditions. Any stable mineral depends on the numerical value for that mineral to exist at certain control condition of temperature, especially at room temperature of 25^0C (298k) and 1atm pressure. The problem involving the distribution of chemical elements and elemental substitutions in rocks during crystallization of magma, from the beginning of crystallization to the end of crystallization was mathematically formulated using Bowen’s and Goldschmidt combined concept and models and analyzed using Metrical matrix and Set notion methods. This research was focused on bridging the gap between Bowen’s and Goldschmidt concepts concerning the problem of elemental substitution and distribution of chemical elements in rocks throughout the time of crystallization from the beginning to the end of crystallization with mathematical foundation such as Metrical matrix and Set notion methods under thermodynamic change. Finally the “mathematical connection” between Bowen’s and Goldschmidt concepts was used in this research for complete explanation of rock forming minerals from the beginning to the end of crystallization and would help the ‘beginners’ especially students of Earth sciences such as Geology, Mineralogy, Petrology and other chemical science such as Geochemistry and Petroleum Geology to have clear understanding of rock forming minerals from the Magma.

Keyword

Goldschmidt, Bowen, Matrix, Set, Thermodynamics.

1. Introduction

The gap between Bowen’s and Goldschmidt concepts concerning the problem of elemental substitution and distribution of chemical elements in rocks throughout the time of crystallization from the beginning to the end of crystallization would be investigated using mathematical context with respect to thermodynamic principles.

Geometrically, several minerals set in matrices and aggregate themselves in coordinate to form rocks and chemically elements of several properties bonded together to form minerals, which by definition is inorganically in composition with distinct chemical composition. Goldschmidt classified these elements according to their class of materials such as siderophile, lithophile and chalcophile elements as well as atmophile elements. But Mendeleev’s (1914) arranged these elements in periodic table according to size and atomic mass. According to modern periodic law, atomic number increases from left to right and from top to bottom of periodic table. Therefore the size and electronegativity of elements increase across the period and the size increases with decreasing electronegativity down the group. This pattern of modern periodic law would be used to explain how ions would enter into the lattice of a growing crystal during crystallization of silicate magma as a function of size and electronegativity. During crystallization of magma, temperature falls, with increasing content of silicon in the magma. This forms two series of reactions (Bowen’s), one by interaction with magma, with the first mineral formed, to form minerals of different chemical composition, but with the same crystallographic structure. This is the case of solid solution in the magma. While the other, the first crystal to form, initially interact with the magma to form a solid with different crystallographic structure, and distinct chemical composition. This is the case of fractional crystallization in the magma.

The term solid solution and fractional crystallization are used simultaneously in this research to define the two series of reactions as stipulated by Bowen’s reaction series.

Mathematically, the formation of rock forming minerals from magma, depends on the numerical values of each of the minerals under thermodynamic change. Any stable materials depend on the numerical values for those materials to exist at that certain ambient temperature, especially at room temperature of 25^0C (298k), at 1atm pressure. The problem involving the distribution of chemical elements and elemental substitutions in rock during crystallization of magma $\mathfrak{M}$, from the beginning of crystallization to the end of crystallization would be postulated to some an extent using combined Goldschmidt and Bowen's concept with respect to electronegativity and radius ratio, which states that, for ions of various electronegativities and sizes to compete for the space in a lattice of a growing crystal;

1. The one with more electronegative would preferentially enter into the lattice of the growing crystal of silicate radical because the nucleus of silicate radical is electronegative provided modern periodic law is obeyed and
2. That ions of closest radius (δ) to the nucleus of formed crystal substitute themselves before the ion of other radius ratio according to Wood (2003).

Bowen (1928) utilized temperature and time to establish relationships among essential rock-forming minerals, while Goldschmidt (1937) used ionic charge and size to explain ionic substitutions in crystals (Diadochi).

1. In a mathematical context, a significant challenge arises as there was no documented mathematical framework complementing the explanation of Bowen's reaction series, which encompasses the continuous (solid solution) and discontinuous (fractional crystallization) reaction series.

2. Both Bowen and Goldschmidt lacked a mathematical connection in their rules, hindering a complete mathematical explanation of rock-forming minerals and their crystallization processes.

3. To achieve a holistic understanding, a need exists to bridge the gap between Bowen's and Goldschmidt's rules with a mathematical foundation to comprehensively explain the formation of rock-forming minerals and their reactions.

This research is focused on bridging the gap between Bowen's and Goldschmidt concepts concerning the "problem of elemental substitution and distribution of chemical elements in rocks throughout the time of crystallization from the beginning to the end of crystallization with mathematical foundation such as Metrical matrix and Set notion under thermodynamic change" with the following objectives;

1. With a mathematical context, Bowen's and the Goldschmidt combined concept would be established using proper computations of numerical values of minerals under thermodynamic conditions which bridge the gap between the two combined rules

2. Mathematical methods such as Metrical Matrix and Set notion would be used to compute and define all the minerals and rocks in a complex plane of magma $\mathfrak{W}$.

2. Literature Review

Bowen's reaction principle, first propounded in 1928 by Norman Bowen, which explains how mineral can respond to changing equilibrium conditions when a magma is cooled, by either a continuous diffusing – controlled exchange of elements with the magma or discontinuous melting of the material [1]

The periodic law was developed independently by Dmitri Mendeleev and Lothar Meyer in 1869. Mendeleev created the first periodic table and was shortly followed by Meyer [2]. They both arranged the elements by their mass and proposed that certain properties periodically reoccur. Meyer formed his periodic law based on the atomic volume or molar volume, which is the atomic mass divided by the density in solid form. Mendeleev's table is noteworthy because it exhibits mostly accurate values for atomic mass and it also contains blank spaces for unknown elements.

Goldschmidt proposed his Classical general rules to explain the distribution of the elements, in which ions of similar size and charge substitute themselves [3].

Ringwood proposed the complementary use of the concept of electronegativity in order to understand the distributions of the chemical elements that could not be explained completely with the Goldschmidtian rules, especially when the minerals being investigated had high percentages of covalent bonding [4].

Bernard J. Wood Modifies Goldschmidt rules 2 and 3, that the site has a preferred radius of Ion (r) which eaters mostly easily, for ions of the same charge, those which are closest in radius to enter most easily, ions which are larger or smaller are discriminated against [5].

Fournier and Rowe, state that silica Geothermomenter works because that solubility of the various silica minerals (Quartz, and chalcedony, S_iO_2) increase monotonically with temperature [6]

Accordind to Balogun Ometere Deborah, Oluwafemi Israel Oluwarotimi and Famakinwa Olamigoke Emmanuel (2024), who reviewed **Adams-Bashforth** method for numerical solution of first order ordinary differential equations and used it to solve first order ordinary differential equation for the field of physical science and Engineering especially the population growth [7]. The Adams -Bashforth methods were designed by John Couch Adams to solve a differential equation modelling capillary action due to Francis Bashfor

Toramaru, A., and Kichise, T., [8], proposed a new numerical experiments to study the influence of different cooling rates and classical nucleation theory parameters on the crystal number density measured under constrained conditions in the laboratory experiments.

3. Mathematical Methods

Mathematical methods are used to resolve the problem of elemental substitution and distribution of chemical elements in rock throughout the time of crystallization from the beginning to the end of crystallization. The mathematical methods include:

a. Matrix method

b. Set Notation Method

3.1 Metrical Matrix

G.V. Gibbs, [9] used metrical matrix to find the equation of a lattice (crystal) plane and its Miller indices as explained below.

$$\text{Where}\quad G=\begin{vmatrix} a^2 & ab\cos\gamma & ac\cos\beta \\ ab\cos\gamma & b^2 & bc\cos\alpha \\ ac\cos\beta & bc\cos\alpha & c^2 \end{vmatrix}$$

Suppose that the unit cell dimensions of a triclinic rhodonite crystal are a = 10.497Å

b =9.797Å, c= 12.185Å, $\alpha = 103.00^\circ, \beta = 108.51^\circ$ *and* γ=82.50° with this information, the metrical matric becomes

$$G=\begin{bmatrix} 110.187009 & 13.423197 & -40.606321 \\ 13.423197 & 95.981209 & -26.853857 \\ -40.606321 & -26.853857 & 148.474225 \end{bmatrix}$$

3.2 Rough set Theory

Shaaban M. Shaaban, Sameh Z. Tawfik [10], rough used set theory to extract volcanic rocks information effectively, a large number of characteristic data must be objectively filtered out. When the best combination of characteristic parameters is achieved, it can be used to identify volcanic rocks precisely. After evaluating many non-linearity computational methods, no further data or previous knowledge were found to be needed for RS theory. The study of geological and volcanic rock information based on RS is a sort of new solution to the mainly geological high-dimensional complex NP (Nondeterministic Polynomial) problems.

1. Information System
2. Indiscernible Relation

3.3 Methods Adopted in this Research

Metrical Matrix

Achuenu and others [11] used the mathematical equation below to calculate the numerical values of all the minerals in rocks with respect to thermodynamics and it represents the Mathematical connection between Bowen's and Goldschmidt concepts.

Therefore for;

$$\sum_{n=r}^{P}\begin{pmatrix} n \\ p \end{pmatrix}[\beta-\alpha]_{n+p}\ (\mathrm{Z}_0)_{\bar{\sigma}}= \binom{n}{p}\ [\beta_{n-p}]\alpha_0\,(\mathrm{Z}_0)_{\bar{\sigma}}-\binom{n}{p}\ [\beta_{n-p}]\ \alpha_1\,(\mathrm{Z}_0)_{\bar{\sigma}}+\ \binom{n}{p}\ [\beta_{n-p}]\ \propto_2(\mathrm{Z}_0)_{\bar{\sigma}}$$

Mathematically, in complex analysis;

$$\mathrm{i}^2 = -1$$

$$\sum_{n=r}^{P}\begin{pmatrix} n \\ p \end{pmatrix}[\alpha+\beta]_{n+p}(Z_0)_{\bar{\delta}} = [\beta_n](Z_0)_{\bar{\delta}} + i\{[\beta_{n-p}\alpha_p](Z_0)_{\bar{\delta}}\} + [\propto_p](Z_0)_{\bar{\delta}}$$

'i' is a complex number in silicate Magma.

$$({}^{\mathrm{n}}\mathrm{c}_{\mathrm{p}},) = \binom{n}{p}$$

$$\binom{n}{p} = \frac{n!}{(n-p)!\,p!}\ \textit{and}\ O! = 1$$

$$\delta\ = \mathrm{o, p, a, m\ and\ f}$$

Where Z_0 is the silicate identity and

$$\eta = \frac{Z^-}{X^+} \text{ (ionic spacies)}$$

‘ η ’ is the strength of the magma $\mathfrak{W}$, which is the ratio of the silicate radical z^- to that of cation x^+ under electrolytic condition and ‘p’ is the recipient cation which depends on ‘ η ’.

$\binom{n}{p}$ Is the coefficient of rock forming minerals and it determines the number of outcomes of each mineral in the melt $\mathfrak{M}(Z_0)_{\bar{\delta}}$.

Where, o, p, a. and m are olivine, pyroxene, amphibole, mica and feldspar

3.4 Anhydrous Melt, $\mathfrak{W}$

Magma with no water content at high temperature, silica deficient and low viscosity.

3.4.1 Mathematical Expression for Mafic Olivine Series: $(\beta_{2-p}\ \alpha_p)\ (Z_0)_{\bar{\sigma}}$

$$\sum_{n=r}^{P}\binom{2}{0}[\alpha+i\beta]_2(Z_0)_{\bar{\sigma}} = 1\lceil \beta_2 \rceil(Z_0)_{\bar{\sigma}} - 2\lceil \beta_1\ \propto_1 \rceil(Z_0)_{\bar{\sigma}} + 1\lceil \propto_2 \rceil\ (Z_0)_{\bar{\sigma}}$$

Given that, $\beta = Magnesium$

$\propto = Iron$

Y = Calcium, Chromium, Nickel or Manganese

$(Z_0)_{\bar{\sigma}} = Si\ O_4$

From equation above, we can rewrite it as;

$$\sum_{n=r}^{P}\binom{2}{0}[\alpha+i\beta]_2\ Si\ O_4 = 1\ Mg_2\ SiO_4 + i\{2[\mathrm{MgFe}]\ SiO_4\} + 1\ Fe_2Si\ O_4$$

$$[\mathrm{Mg+iFe}]_2\ \mathrm{Si\ O_4} = \mathrm{Mg_2\ Si\ O_4} + \mathrm{i[Mg\ Fe]Si\ O_4} + \mathrm{i[MgFe]SiO_4} + \mathrm{Fe_2\ Si\ O_4}$$

$$\sum_{n=r}^{P}\binom{2}{0}[\mathrm{Mg+iFe}]_{\mathrm{n+p}}\ \mathrm{Si\ O_4} = \text{Forsterite+i[Hyalosiderite]+i[Hortonolite]+Fayalite}$$

According to Achuenu and others (2025), Mg2-pFepSiO4 is an ‘Olivine series’ formula that can be used to calculate all olivine minerals from the melt $\mathfrak{W}$, where p is an integer and ranges from 0 to 2 in olivine crystals. At p equals to zero, 100% Forsterite (Fo) crystallizes with chemical formula of Mg2SiO4 and at p equals to 2, Forsterite (fo) disappears and 100% of pure Fayalite (Fa) crystallizes with chemical formula of Fe2SiO4(s) .Therefore forsterite, Hyalosiderite , Hortornolite and fayalite are the primary olivines in the mafic olivine series.

3.4.2 Expected Mathematical Expression for Mafic Pyroxene Series: $[\beta_{2-p}\ \alpha_p\ ](Z_0)_{\bar{p}}$

$$\sum_{n=r}^{P}\binom{2}{0}\ [\beta + \alpha]_{2+p}\ \mathrm{Si_2O_6} = 1\ \mathrm{Mg_2\ Si_2O_6} + \mathrm{i[2(MgFe)Si_2O_6]} + \mathrm{Fe_2\ Si_2O_6}$$

$$\sum_{n=r}^{P}\binom{n}{p}[\beta+\alpha]_{n+p}\ Si_2\ O_6 = Mg_2\ Si_2\ O_6 + i[Mg\ \mathrm{Fe}]\ Si_2\ O_6 + i[\mathrm{MgFe}]Si_2\ O_6 + Fe_2\ Si_2\ O_6$$

Pyroxene = Enstatite +Magnesio-hypersthene + Ferro-hypersthene + Ferrosilite

$Mg_{2-p}Fe_pSi_2O_6$ is a pyroxene series, where p range from 0 to 2 in pyroxene. At p equals to zero, 100% Enstatite (En) crystallizes with chemical formula of $Mg_2Si_2O_6$ and at p equals to 2, Enstatite disappears and 100% of pure Ferrosilite crystallizes with chemical formula of $Fe_2Si_2O_6$

Therefore Enstaite, Hypersthene, Eulite and ferrosilite are the primary pyroxenes in the mafic pyroxene series.

3.5 Hydrous Melt

Magma with water content at low temperature, silica saturation and high viscosity

3.5.1 Expected Mathematical Expression for Mafic Amphibole, $[\beta_{7-p} \propto_p](Z_0)_{\bar{a}}2$

$$(Z_0)_{\bar{a}} = \mathrm{Si_4O_{11}(OH)^{-7}}$$

$$(Z_0)_{\bar{a}}2 = Si_8O_{22}(OH)_2$$

$$\sum_{n=r}^{P}\binom{7}{0}[\beta+\alpha]_7(Z_0)_{\bar{a}}2 = 1[\beta_7](Z_0)_{\bar{a}}2 + 7i[\beta_6 \propto](Z_0)_{\bar{a}}2 + 21[\beta_5 \propto_2](Z_0)_{\bar{a}}2 +$$

$$30i[\beta_4 \propto_3](Z_0)_{\bar{a}}2 + 30[\beta_3 \propto_4](Z_0)_{\bar{a}}2 + 21i[\beta_2 \propto_5](Z_0)_{\bar{a}}2 + 7[\beta_1 \propto_6](Z_0)_{\bar{a}}2 + 1i[\propto_6](Z_0)_{\bar{a}}2$$

$$\sum_{n=r}^{P}\binom{7}{0}[Mg+Fe]_7Si_8O_{22}(OH)_2=1[Mg_7]Si_8O_{22}(OH)_2+7i[Mg_6Fe_1]Si_8O_{22}(OH)_2+21[Mg_5Fe_2]Si_8O_{22}(OH)_2+$$

$$30i[Mg_4Fe_3]Si_8O_{22}(OH)_2+30[Mg_3Fe_4]Si_8O_{22}(OH)_2+21i[Mg_2Fe_5]Si_8O_{22}(OH)_2+7[MgFe_6]Si_8O_{22}(OH)_2$$
$$+1i[Fe_6]Si_8O_{22}(OH)_2$$

Then;

$$[\beta_{7-p}\propto_p](Z_0)_{\bar{a}}2 = [(Mg7\text{-}pFep)(Si8O22(OH)2]$$

According to Achuenu and others (2025), ***[(Mg$_{7-p}$Fe$_p$)(Si$_8$O$_{22}$(OH)$_2$]*** is a chemical formula for an Amphibole series, when p ranges from 0 to 7. At p equals to zero, 100% of Kupfferite (Ku) with chemical formula of $Mg_7Si_8O_{22}(OH)_2$ crystallizes. At p equals 7, Kupfferite disappears, and 100% pure Grunerite (Gr) crystallizes with chemical formula of $Fe_7Si_8O_{22}(OH)_2$

3.5.2 Mathematical Expression for Mafic Mica Series: $[\beta_{6-p} \propto_p](Z_0)_{\bar{m}}2$

The crystal of amphibole interacts with the magma, so that the **$Si_4O_{11}(OH)$** of amphibole links to cation is being hydrolyzed in the presence of excess water in the magma as shown in equation below

$$Si_4O_{11}(OH)(aq) + 0H_{(aq)} \longrightarrow Si_4O_{10}(OH)_2(aq)$$

$$(Z_0)_{\bar{a}}2 = Si_4O_{20}(OH)_4$$

$$\sum_{n=r}^{P}\binom{6}{0}[Mg+iFe]_6Si_4O_{20}(OH)_4 = 1[Mg_6]Si_4O_{20}(OH)_4 + 6i[Mg_5Fe_1]Si_4O_{20}(OH)_4 +$$
$$15[Mg_4Fe_2]Si_4O_{20}(OH)_4 + 20i[Mg_3Fe_3]Si_4O_{20}(OH)_4 + 15[Mg_2Fe_4]Si_4O_{20}(OH)_4 + 6i[Mg_1Fe_5]Si_4O_{20}(OH)_4 + 1[Fe_6]Si_4O_{20}(OH)_4$$

Mafic Mica = Phlogopite + i[Magnesio-biotite]+ Biotite +i[Ferro-biotite]+ Lepidomelane

$$[\beta_{6-p}\ \alpha_p](Z_0)_{\bar{m}}2 = [Mg_{6-p}Fe_p][Si_8O_{20}(OH)_4]$$

According to Achuenu and others (2025), ***[Mg$_{6-p}$Fe$_p$][Si$_8$O$_{20}$(OH)$_4$]*** is a biotite series, as p ranges from 0 to 6. At 'p' equals to zero, 100% of phlogopite (Ph) crystallizes, and at 'p' equals to 7, Phlogopite (Ph) disappears and 100% pure Lepidomelane (Lp) crystallizes with a chemical formula of $K_2Fe_6Al_2Si_6O_{20}(OH)_4$.

3.5.3 Expected Mathematical Expression for Plagioclase Series: $(\Upsilon_{m-p}X_p)(Q_{x-y}N_y)\mathcal{W}_i$

$$(\gamma+X)_{x+y}(Q+\mathcal{N})_{m+p}\mathcal{W}_i = (\gamma_{m-p}X_p)(Q_{x-y}\mathcal{N}_y)\mathcal{W}_i$$

$$[X_{m-p}\Upsilon_p]Zi=[X_{1-p}\Upsilon_p](Z_0)_{\bar{f}}2$$

$$= [X_{m-p}\Upsilon_p](Z_0)_{\bar{f}}2$$

$$(Z_0)_{\bar{f}}2 = S_iO_4$$

$$(Z_0)_{\bar{f}}2 = S_{i\,4}O_8$$

$$\Upsilon\text{−Position} = \Upsilon + X$$

$$[X_{m-p}\Upsilon_p]\ (Z_0)_{\bar{f}}2 = [X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$$

For p = 1 and y =1

$$[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i = [X_{1-1}\gamma_1][Q_{4-1}\mathcal{N}_1]\mathcal{W}_i$$

$$[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i = [\gamma_1][Q_3\mathcal{N}_1]\mathcal{W}_i$$

$$\gamma_m[Q+\mathcal{N}]_{x+y} + [x]_p(Q+\mathcal{N})]_{x+y} = [X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$$

$\gamma_m[Q+\mathcal{N}]_{x+y} + [x]_p(Q+\mathcal{N})]_{x+y}$ + is called "PLAGIOCLASE SERIES"

$\gamma_m[\mathcal{Q}+\mathcal{N}]_{x+y}+[x]_p(\mathcal{Q}+\mathcal{N})]_{x+y} = \mathrm{CaAl_2Si_2O_8 + NaAlSi_3O_8}$

Given that;

$\gamma_1[Q_2N_2]0_8 = \mathit{Basic\ plagioclase}(\beta_a f)$

$X_1[Q_3N]0_8 = \mathit{Alkaline\ feldspar}(\alpha_c f)$

$[\beta_2](\mathrm{Z}_0)_{\bar{o}}=\mathit{Basic\ mineral}\ (\beta_a m)$

$[\alpha_2](\mathrm{Z}_0)_{\bar{o}}=\mathit{Acidic\ mineral}\ (\alpha_c m)$

Therefore in Matrix representation we have;

$$\begin{bmatrix} \beta_a P \\ \beta_a M \end{bmatrix} + \begin{bmatrix} \alpha_c P \\ \alpha_a M \end{bmatrix} = \begin{bmatrix} f \\ M \end{bmatrix}$$

$$\begin{bmatrix} \gamma_1[\mathcal{Q}_2\mathcal{N}_2 0_8 \\ [\beta_n](Z_0)_{\bar{o}} \end{bmatrix} + \begin{bmatrix} \gamma_1[\mathcal{Q}_2\mathcal{N}_2 0_8 \\ [\alpha_p](Z_0)_{\bar{o}} \end{bmatrix} = \begin{bmatrix} F \\ M \end{bmatrix}$$

As, $\alpha\beta \to 0$

3.5.4 Set Notation

Set notation is a mathematical parameter used in this research to partition minor elements of small concentrations in part per million (ppm), such as, transition elements, and Rare Real Earth Elements, according to their compatibility and incompatibility in the magma. The compatibility and the incompatibility, of these minor elements depend on the ratio of the concentration of minor elements in the solid (C_s) to the concentration of the minor elements in liquid in the magma during crystallization. The ratio of the concentration of the concentration of minor elements in solid to that of the minor elements in Liquid is constant(K_n). Therefore, this can be mathematically expressed as

$$K_n = \frac{C_s}{CL}$$

Where; C_s=*concentration in solid*

CL=*Concentration in Liquit*

K_n= Constant.

Therefore, if the ratio of the concentration of trace elements or REE in solid (C_s) to that of the liquid is greater one, $K_n > i$, then the trace or REE is compatible, and can be partitioned into the solid phase. In this case, for compatible trace or REE, k_n such that;

$$\frac{C_s}{CL} > 1$$

If the ratio of concentration of trace elements, transition or REE in solid (C_s) to that of the Liquid (CL) *is less tahn one,* $k_n < 1$ then the minor elements is incompatible and can be partitioned along the liquid phase in this case, for incompatible minor elements, $K_n < 1$, *such that*

$$\frac{C_s}{CL} < 1$$

It pertinent to note that, those minor elements that partitioned along the fluid phase, such as hydrous phase or gaseous phase are mostly High Field strength Elements (HFSEs), because they have small ionic radius and large ionic charge. They from the major gemstones and ore deposit of the world. They are also called small ion Lithophile elements.

The mathematical equations above can be represented using set Notation

$$K_d = [K_d\ \text{miner al(1 m the rock)}]+[\mathfrak{M}_{inz}] + [\mathfrak{M}_{in}\,\mathfrak{Z}]$$

4. Results and Analyses of the Research.

4.1 Introduction

In a new model of crystallization in magma, Toramaru, A., and Kichise, T., [12], used a new numerical experiments to study the influence of different cooling rates and classical nucleation theory parameters on the crystal number density measured under constrained conditions in the laboratory experiments and identifies that, the nucleation rate as the main parameter that controls the formation of crystal. In this condition it is interesting that the crystal growth rate is inversely proportional to the time, even if the diffusion limited growth is inversely proportional to the square root of time. In other words, the nucleation process is controlled by an interplay between decompression and diffusion rates.

Andrew, B.J., and Befuse, K, S, [13], developed a numerical model which is supersaturation nucleation and growth of plagioclase that predicts the nucleation and growth of plagioclase crystals in a decompressing magma as a function of time and that growth rates are used to grow the existing crystals, where nucleation adds new crystals.

Supersaturation is the driving force of the crystallization, the rate of nucleation and growth is driven by the existing supersaturation in the solution. Depending upon the conditions, either the nucleation or growth may be predominant over the other and as a result, crystals with different sizes and shapes are obtained. Once the supersaturation is exhausted, the solid-liquid system reaches equilibrium and the crystallization is complete, unless the operating conditions are modified from equilibrium so as to supersaturate the solution again. The crystal growth is the subsequent growth of nuclei that succeed in achieving the critical cluster size as shown Figure 5. Nucleation and growth continue to occur simultaneously while the supersaturation exists

Under electrolytic condition with respect to thermodynamic change, there are two principal parameters in which crystals are formed from the electrolytic melt $\mathfrak{W}$, and the formation of these two principal parameters depend on two principal factors controlling the growth of the crystal and these principal factors are the;

1. size of growing crystal
2. equilibrium temperature.

The point at which lattice of growing crystal $\mathbf{Z_0}$, remains at equilibrium with the melt $\mathfrak{W}$, without any external constraints is called “***Critical Point of the Growing Crystal, Z_0***

If the critical point of the growing crystal in the complex plane of magma $\mathfrak{W}$, ABCD, has been established during the growth of crystals, considering the **size** and the **equilibrium temperature** of the growing crystal, then crystals would form and remain stable when the equilibrium is above the **critical point** between the **melt** $\mathfrak{W}$, and the **crystal** Z, but when the equilibrium is below the **critical point** between the **crystal** Z, and the **melt** $\mathfrak{W}$, under thermodynamic change, then the crystal Z, would be unstable and resolve back to the melt $\mathfrak{M}$.

For ions or atoms to enter into the lattice of forming crystal, the following rules by Wood (2003) must be satisfied

1. When various ions in the Recharge magma $\mathfrak{M}$ are competing for space in the lattice of a growing crystal, the ions with the smallest sizes would eventually enter into the lattice of the growing crystal Z_0.
2. When various ions in the recharged magma $\mathfrak{M}$ with different sizes and similar charge are competing for substitution in already formed crystal Z, then the ion with the closest radius ratio to the nucleus of the crystal would go into substitution provided that the difference between the size of the ions involve in the substitution is not differ by 15% and the concentration of the ions in the melt is saturated enough to enter into substitution, otherwise ions of higher concentration in the melt could go into substitution.

The probability of certain atoms or ions to enter into the lattice of growing crystal and a formed crystal Z, depends on the size, charge and the electronegativity of the atoms in the periodic table as shown in Table 1 and Figure 1 below;

Table 1. Showing the Relative bonding, Ionic radius, Ionic charge, Ionization and Electronegativity of elements

Element	Electronegetivity	Ionization Energy	Ionic radius	**Ionic Charge**	**Total Relative Bonding**
Sodium	0.90	5.133	0.97	**+1**	**100**
Potassium	***0.80***	***4.339***	***1.33***	***+1***	***90***
Magnesium	1.20	14.970	0.66	+2	202
Calcium	1.0	11.820	0.99	+2	200
Aluminium	1.50	28.31	0.51	+3	300
Cromium	1.50/1.40	15.700	0.63	+3	321
Manganese	1.60	32.100	0.80	+2	174
Iron	1.65	16.240	0.644	+2	174
Iron	1.80		0.74	+3	
Nickel	1.8/1.7	18.130	0.69	+2	1.97
Silicon	1.80	44.950	0.42	+4	380
Cobalt	1.80/1.70	17.300		+2	183
Chlorine	3.0		1.81	-1	
Fluorine	4.0		1.33	-1	
Oxygen	3.50			-2	384

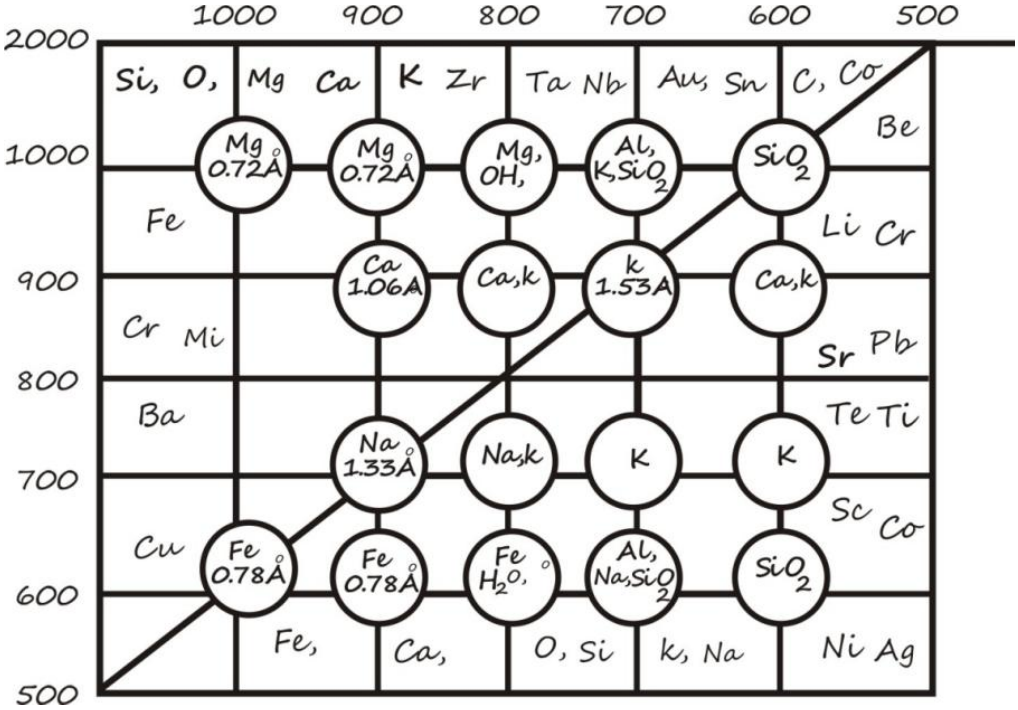


Figure 1. Nucleation of Atoms with Respect to sizes and charges in the magma $\mathfrak{W}$

The two principal parameters in which crystals are formed from Electrolytic magma $\mathfrak{W}$ are;

1: Lattice of '**Growing** 'crystal **(Z_0)**.

Lattice of a growing crystal is the critical point in the electrolytic melt $\mathfrak{W}$, in which the size of the growing of the crystal is at equilibrium with the equilibrium temperature, such that any infinitesimal change in temperature would cause the growing crystal to

I. be stable, when the critical point is above equilibrium temperature.
II. resolve back to the recharged melt, when the critical point is below equilibrium temperature

2: **Lattice of 'formed' crystal (Z).**

This is the point in the recharged melt $\mathfrak{W}$, in which the size of the crystal formed is above the equilibrium temperature and at this point Gibbs free energy is attained, such that any infinitesimal change in temperature would not cause the formed crystal to resolve back to the recharged melt, therefore it requires a great amount of heat energy to break such bond between the atoms holding each other in the formed crystal.

With the heterogeneous nucleation, new minerals nucleate by taking advantage of the structure of an existing mineral. Some of the energetic problems associated with homogeneous nucleation are avoided. If an existing mineral has a surface or structure that is similar to a new mineral, the existing mineral can serve as the nucleus for growth. The need to form embryos is largely eliminated. Once nuclei are stable, growth must progress by adding atoms/ions to the surface of the crystal. This faces kinetic problems similar to nucleation.

Under thermodynamic change and electrolytic condition, the formed crystal Z_1, in the melt $\mathfrak{W}$, becomes unstable and spontaneous. At this point, equilibrium between the melt $\mathfrak{W}$ and the formed crystal Z, is below the critical point with higher entropy change ($\Delta s > 0$) and lower Gibbs free energy ($\Delta G < 0$) of the system containing the crystal Z_1, and in this spontaneous process, crystal begins to change from one form to the other by replacement of atoms of similar size and charge until equilibrium is reached and crystal Z_2, is formed. When equilibrium is above the critical point with lower entropy ($\Delta s < 0$) change and higher Gibbs free energy ($\Delta G > 0$), provided that there is no change in thermodynamic, and if equilibrium is prevailed at this point, then crystal Z_2 formed, would be stable with the melt $\mathfrak{W}$ When atoms of similar size and charge substitute themselves in the lattice of formed crystal Z_1 under spontaneous process, to form another crystal Z_2 of the same structure, it is called ***Isomorphous*** substitution reaction.

1. When atoms of different sizes and the same charge are competing for substitution, according to Wood in the lattice of a formed crystal, Z, the one with closest radius ratio to the atom or ion in the site would eventually go into substitution in preference to the one lower or high in radius provided that, the difference in their radius is not differed by 15% and if the concentration of the substituting crystal is saturated enough in the magma containing the crystal.
2. When atoms of different charges and similar size are competing in the lattice of a formed crystal, according Wood, the one with closest charge to the atoms in the lattice would preferentially go into substitution given that concentration is saturated enough in the melt containing the crystal.

4.1.1 Strength of Magma (η) under Thermodynamic Condition.

Under electrolytic condition, at a specified temperature, pressure and concentration, the strength of magma (η) is the ratio of atomic nucleation of anion or radical $\mathbf{Z_0}$, to that of the closest cation $\mathbf{X^+}$ in the electrolytic magma, $\mathfrak{W}$. In this case the closest cation is the one in which its electronegative is close to the anion in question in preference to other cations in the electrolytic magma, $\mathfrak{W}$.

Therefore;

$$\eta = \frac{Z_0}{X^+} \text{ (ionic species)}$$

4.1.2 Stability of Minerals in an Electrolytic Magma

The stability of crystals are defined according to the characteristics roots of the equations (Adams, and Bashforth). The conditions of stability of minerals in rock during crystallization According to Adams and Bash forth include the following:

1. Methods that satisfy the root condition and have Z = 1 or 2 as the only root of the characteristics equation of magnitude one or more are called strongly stable.

2. Methods that satisfy the root condition and have more than one distinct root with magnitude one or more are called weakly stable.

3. Methods that do not satisfy the root condition are called unstable.

Considered the conditions for stability mentioned above, the algebraic sum of the constituent elements in each mineral is equal to zero with respect to their distinct characteristics roots, and stability depends on the Gibbs free energy of the mineral.

Therefore crystals are strongly stable but their Gibbs free energy is a function of the available cation at that time -during crystallization. This means that Gibbs energy determines the stability of the minerals when temperature drops further. This implies that minerals rich in magnesium are strongly stable because it has one root that is greater one, but it is metastable, because minerals rich in magnesium are not in their minimum Gibbs energy state. Those minerals rich in ion are strongly stable, and are in the minimum Gibbs free energy state and has the characteristic root not greater than one. The equal proportion of these two elements in minerals means that they are weakly stable (metastable), because it contains characteristics two roots.

According adams- Bash and adams –Moulton, a method is convergent if the solution to difference equation approaches to the solution of differential equation as the step size approaches zero as given as;

$$\text{Lim } h \longrightarrow 0<1<N\ [w_i - y(t_i)] = 0$$

A stable method is the one whose result depends continuously on the initial data.

Given that $\beta_n(Z_0)=O$ is at stable equilibrium with the silicate melt, but not at its minimum Gibbs free energy which is numerically equals to zero, $\Delta G = O$ but for a free minimum Gibbs energy, $\alpha_p(Z_0) = O$ at $\Delta G > O$ is at equilibrium with the silicate the silicate melt. Therefore,

- β_n (Z_0), a metastable equilibrium, with $\Delta G=O$
- α_p (Z_0) a stable equilibrium, with $\Delta G>O$

This implies that,

1. Minerals rich in magnesium are strongly stable because it has one root that is greater than one, but it is metastable because minerals rich in magnesium are not in their minimum Gibbs free energy state in the presence of free silica.
2. Minerals rich in iron are strongly stable and are in their minimum Gibbs free energy state in the presence of free silica and has the characteristics one root greater than one.

4.2 Analysis of Results

The mathematical methods used to analyze these minerals in the rocks and classify them according to their genetic origin include;

a. Matrix method

b. Set Notation

4.3 Application of Metrical Matrices to Rock Forming Minerals

The use of matrices to resolve some problems concerning chemical distribution and elemental substitution of chemical elements in rocks becomes very important in this research.

In mathematics, a matrix is a rectangular array of numbers, symbols or expressions, arranged in rows and columns, these numbers are called the elements or entries of the matrix. Matrices have wide applications in engineering, physics, economics, and statistics as well as in various branches of mathematics and even geology.

To resolve these problems using matrix method, minerals set in matrix to form rock, such minerals will form simultaneously at the same temperature under isobaric condition and attain a particular Gibb's energy to maintain stability. For a heterogeneous rock to form, the rock classifiers, feldspars will form simultaneously with the mafic minerals at an approximate temperature under isobaric condition.

Given that;

$$\beta_a P = \textit{Basic Plagioclase}$$

Basic Feldspar = Sequence and initial value

$$\alpha_c P = \textit{Acidic Plagioclase}$$

Acidic Feldspar = Subsequence and modified value

$$\beta_a M = \textit{Basic mafic}$$

Basic mafic = Sequence and initial value

$$\alpha_c M = \textit{Acidic mafic}$$

Acidic mafic = Subsequence and modified value and such that;

Basic Plagioclase+Basic mafic=Basic (1)
Basic mafic=basic olivine, *basic pyoxene and opaque minerals*
Basic *Plagioclase=Basic Anorthite*, *Basic Bytownite*, *basic labradiorite*
Acidic Plagioclase+Acidic alkali=Felsic (2)
Acidic *plagioclase=Oligoclase*
Acidic alkali = albite
Acidic plagioclase+Acidic alkali=Felsic (3)
*plagioclase=Oligoclase and andesin*e

Alkaline=Albite, *orthoclase and microdine*

Alkali+Acidic=Acidic (4)
Alkali=albite, *orthoclase and microcline*

Acidic=acidic quartz, *acidic olivine and acidic pyroxene*

Acidic+Basic=Mafic (5)

Then,

$$\beta_a P + \alpha_c P = \mathcal{F} \quad (6)$$

$$\beta_a M + \alpha_c M = \mathscr{M} \quad (7)$$

If $\gamma_m [Q + \mathcal{N}]_{x+y} + [x]_p (Q + \mathcal{N})_{x+y} = Felsic$

$$[(\beta)_n + (\alpha)_p] Z_0 = Mafic$$

Such that;

$$\gamma_m [Q + \mathcal{N}_{x+y} + [x]_p (Q + \mathcal{N})]_{x+y} = 0 \quad (8)$$

$$[(\beta)_n + (\alpha)_p] Z_0 = 0 \quad (9)$$

$As\ \beta\alpha \to 0$

Using factor theorem, to evaluate the simultaneous equations above. In algebra, the factor theorem is a theorem linking factors and zeros of polynomial. It is a special case of the polynomial remainder theorem.

The factor theorem states that a polynomial has a factor, if and only if (i.e.; is a root). Then to resolve the simultaneous equations (8) and (9) above, then range from 0 *to* ∞ *where* $\{p{:}y\} \leq x$, *m*

For feldspars;

Given that;

$$\Upsilon_m\ [Q+N]_{x+y} + [x]_p (Q+N)]_{x+y} = \left[\mathrm{X}_{1-p} \Upsilon_p\right] [Q_{4-y} N_y]\ W_i$$

$$[\mathrm{Y}_{1-\mathrm{p}}\ \mathrm{X}_{\mathrm{p}}] \left[\ \mathrm{Q}_{4-\mathrm{y}} \mathrm{N}_{\mathrm{y}}\right] 0_8 = 0,$$

At p =0 and y = 1

T*hen Z,* [(***m,p***)**: (*x,y*)**]***−position***

[m, p]- Position = [1, 0]

[x, y]- Position = [4, 1]

$$\left[\gamma_{1-p} X_p\right][Q_{4-y} \mathcal{N}_y]0_8 = \gamma_1 (Q_3 N_1)$$

At p =1 and y = 2

T*hen Z,* [(***m,p***)**: (*x,y*)**]***−position***

[m, p]- Position = [0, 1]

[x, y]- Position = [4, 2]

$$[\Upsilon_{1-p} X_p] \left[Q_{4-y} N_y \right] 0_8 = X_1 (Q_2 N_2)$$

$$\Upsilon_m + [x]_p (Q+N)]_{x+y} = [\Upsilon_1 (Q_2 N_2) + X_1 (Q_3 N_1)]0_8$$

$$[\gamma_1 Q_2 \mathcal{N}_2 + X_1 Q_3 \mathcal{N}]0_8 = [1, 0+2, 2] + [0, 1+3, 1] \quad (10)$$

$$[(\gamma + X) Q_5 \mathcal{N}_3]0_{16} = [(1,1) + (5,3)]16 \quad (11)$$

For Mafic

Given that;

$$(\beta + \alpha)_{n+p} Z_0 = [\beta_{2-p} \alpha_p] Z_0$$

as $n = 0 \longrightarrow 2$, *and* $p \leq n$

$$\therefore [\beta_{2-p} \alpha_p] Z_0 = 0 \quad (12)$$

Using factor theorem;

For n = 2

(2- p)p = 0

P = 0

Then Z,*[*n,p*]*−position

[n: p]- Position = [2, 0]

$$[\beta_{2-p} \alpha_p] Z_0 = [\beta_2] Z_0$$

$2 - p = 0$

P = 2

n = 0

Then Z,* [*n,p*]*−position

[n: p]- Position = [0, 2]

$$[\beta_{2-p} \alpha_p] Z_0 = [\alpha_2] Z_0$$

$$\therefore [\beta_{2-p} \alpha_p] Z_0, at[2,0] = [\alpha_2] Z_0 \quad (13)$$

$$[\beta_{2-p} \alpha_p] Z, \ at \ [0, 2] = [\beta_2] Z_0 \ (14)$$

put $[\beta_2]Z_0$ and $[\alpha_2]Z_0$ in equation

$$[\beta_{2-p} \alpha_p] Z_0 = [2, 0] + [0, 2]$$

$$[\beta_2 + \alpha_2] Z_0 = [2, 0] + [0, 2] \ (15)$$

β=basic, so that $[\beta_2]Z_0$ *is basic mineral*

α=acidic, so that $[\alpha_2]Zo$ *is acidic mineral*

$$[\beta + \alpha] = [2, 2] \ (16)$$

Using Cauchy’s definition of sequence;

$\mathscr{E} > 0$ and $\delta < \mathscr{E}$

For, $\delta = 1$ for unit sphere, therefore, $\mathscr{E} = 3.142$.

$\eta_z = \varepsilon_z$ (17)

$\mathscr{E} = 3.142$

$\delta = 1$unit

$\eta_z = \mathscr{E}_z - 1$ (18)

Sequentially, a sphere $\mathscr{E}$ with a unit length δ can be constructed using Cauchy definition of as shown in Tables 2 and 3 below;

Table 2. Sequence and subsequence of a sphere, $\mathscr{E} = \pi$

Number of times	**Sequence $\mathscr{E} = \pi$**	**Subsequence η**	**Difference**
	$\mathscr{E}_z$	$\eta_z = \mathscr{E}_z - 1$	$\mathscr{E}_z - \eta_z$
1	$[\beta_2]Z_0$	$[\alpha_1]Z_0$	**1, 0**
2	$[\beta_{2-p}\alpha_p]Z$		**1, 2**
3	$[\alpha_2]Z_0$		**2, 0**
Total		$[\alpha_2]Z_0$	**0, 2**

Table 3. Sequence and subsequence of a sphere, $\mathscr{E} = \pi$

Number of times	**Sequence $\mathscr{E} = n$**	**Subsequence η**	**Difference**
	$\mathscr{E}_z$	$\eta_z = \mathscr{E}_z - 1$	$\mathscr{E}_z - \eta_z$
1	**Basic**	**Acidic**	
2	**Intermediate**	**intermediate**	
3	**Acidic**	**Basic**	
Total		**Acidic**	

4.3.1 Metrical Matrix for Mafic Olivine Minerals and Feldspars: $(\beta_{2-p}\ \alpha_p)(Z_0)_{\bar{o}}$

Olivine and feldspar in this category of Matrix series are called primary olivine and feldspar in the mafic and felsic magma complexes because they are the first derivatives of olivine and feldspar from the magma $\mathfrak{W}$ and these require simple substitutions of ions of the same charge and similar size according to Goldschmidt.

Then set equations (10) and (15) above, in matrices to form equations (19) below.

$$[\Upsilon_1(Q_2 N_2)]0_8 + X_1(Q_3 N)0_8 = [(1,\ 0],[2,\ 0)] + [(0,\ 1],[3,\ 1)]$$

$$[\beta_2](Z_0)_{\bar{o}} + [\alpha_2](Z_0)_{\bar{o}} = [2,0] + [0,2]$$

$$\begin{bmatrix} \Upsilon_1(Q_2 N_2)0_8 \\ [\beta_2](Z_0)_{\bar{o}} \end{bmatrix}^{,}_{,} + \begin{bmatrix} X_1(Q_3 N)0_8 \\ [\alpha_2](Z_0)_{\bar{o}} \end{bmatrix} = \begin{bmatrix} [0,\ 1],[2,\ 2] \\ [2,\ 0] \end{bmatrix}^{,}_{,} + \begin{bmatrix} [1,\ 0],[3,\ 1] \\ [0,\ 2] \end{bmatrix}$$

$$\begin{bmatrix} \Upsilon_1(Q_2 N_2)0_8 \\ [\beta_2](Z_0)_{\bar{o}} \end{bmatrix} + \begin{bmatrix} X_1(Q_3 N)0_8 \\ [\alpha_2](Z_0)_{\bar{o}} \end{bmatrix} = \begin{bmatrix} [0,\ 1],[2,\ 2] \\ [2,\ 0] \end{bmatrix} + \begin{bmatrix} [1,\ 0],[3,\ 1] \\ [0,2] \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} [(\Upsilon_1 + X_1)Q_5 N_3]0_{16} \\ [\beta_2 + \alpha_2](Z_0)_{\bar{o}} \end{bmatrix} = \begin{bmatrix} [1,\ 1],[5,\ 3] \\ 2, \qquad 2 \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} [0,1],[2,2] \\ [2,0] \end{bmatrix} + \begin{bmatrix} [1,0],[3,1] \\ [0,2] \end{bmatrix} = \begin{bmatrix} [1,1],[5,3] \\ [2],[2] \end{bmatrix} \quad (21)$$

$\gamma_1[Q_2N_2]0_8 = \textit{Basic plagioclase } (\beta_a f)$

$X_1[Q_3N]0_8 = \textit{Alkaline feldspar } (\alpha_c f)$

$[\beta_2](Z_0)_{\bar{o}} = \textit{Basic mineral } (\beta_a m)$

$[\alpha_2](Z_0)_{\bar{o}} = \textit{Acidic mineral } (\alpha_c m)$

$$\begin{bmatrix}\beta_a f\\ \beta_a m\end{bmatrix}+\begin{bmatrix}\alpha_a f\\ \alpha_a m\end{bmatrix}=\begin{bmatrix}f\\ m\end{bmatrix}$$

$$\begin{bmatrix}\gamma_1[Q_2\mathcal{N}_2]0_8\\ [\beta_2](Z_0)_{\bar{o}}\end{bmatrix}+\begin{bmatrix}\gamma_1[Q_2\mathcal{N}_2]0_8\\ [\beta_2](Z_0)_{\bar{o}}\end{bmatrix}=\begin{bmatrix}f\\ m\end{bmatrix} \quad (22)$$

$$\begin{bmatrix}Ca_1[Al_2Si_2]0_8\\ [Mg_2]SiO_4\end{bmatrix}+\begin{bmatrix}Na_1[AlSi_3]0_8\\ [Fe_2]SiO_4\end{bmatrix}=\begin{bmatrix}(Ca_1Na_1)[Q_5\mathcal{N}_3]0_{16}\\ [MgFe]SiO_4\end{bmatrix} \quad (23)$$

$$\begin{bmatrix}Anorthite\\ Forsterite\end{bmatrix}+\begin{bmatrix}Albite\\ Fayalite\end{bmatrix}=\begin{bmatrix}Anorthoclase\\ Olivine\end{bmatrix}$$

$[Basalt]+ \quad [Rhyolite] = [\ Andesite]$

$[basic\ basalt]+[acidic\ Rhyolite]=[Intermediate]$(24)

if pl>α, then then the intermediate rock is Andesite

$[Basalt]+[Dacite]$ = Andesite(25)

if pl <α, then the intermediate rock is Trachyte

$[Basalt]+[Ryolite]=Trachyte$ (26)

if pl=α , then the Intermediate rock is the Hybrid mogonite

$[Basalt]+[Rhyolite]=hybrid\ Monzonite$(27)

Given that;

$\mathcal{E}$ = $[Mg+iFe]_2\ Si\ O_4$

$\mathcal{E}_z = Mg_2\ Si\ O_4$,

$\eta_z = Fe_2\ Si\ O_4$

Adam-Bashforth method of difference equation to the solution of differential equation as the size (δ) of growing embryos (Z_0) approach the critical growth radius (δ_c) to form a stable crystal nuclei (Z) and Cauchy's sequence and subsequence are used to analyze olivine group, using the two end members of Olivine series as shown Table 4;

Mg_2SiO_4 = Initial value and the sequence

$Fe_2\ Si\ O_4$, = Modified value and the subsequences

Therefore, λ_η represents the roots of the equation*s* $[\beta^{2-p\propto p}](Z_0)\bar{\delta}$ for Olivine minerals and $[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

Therefore;

η = 2 for Olivine and η = 2 for Feldspar

Then;

$\lambda_2 = (2, 2)$ for Olivine and $\lambda_2 = (1, 1)$ for Feldspar

Table 4. The two End members of Olivine group and Feldspar group using Adams-Bashfort and Cauchy sequence

Adams-Bashfort	**Initial value**		**Modified value**		**Root**	
$\lim_{\eta\to 0, 0\le p\le n} max\ \lvert\eta_z-\varepsilon_z\rvert=\eta$	$\mathcal{E}_z$ = n		η_z = p		λ_2	λ_2
Chemical formula	Mg_2SiO_4	$Ca_1[Al_2\ Si_2]0_8$	Fe_2SiO_4	$Na_1[AlSi_3]0_8$	2, 2	0.2, 1
Geologic name	Forsterite	Anorthote	Fayalite	Albite	2, 2	0.2, 1
Cauchy's sequence	Sequence	Sequence	subsequence	Subsequence	First	first
Lyapunov (1992)	Stable	Stable	Stable	Stable	Neutral	Neutral

Classification of mafic olivine

- **Olivine class** = 1[basic '***<u>Olivine</u>***'] + [intermediate '***<u>Olivine</u>***'] + i[acidic ***<u>Olivine</u>***].
- **Olivine Minerals** = basic '***<u>Forsterite</u>*** '+ i[intermediate '***<u>Olivine</u>*** '] + acidic '***<u>Fayalite</u>***'
- **Olivine rocks** = (olivine'**Basalt**')+(oline'**Granite**')+i(olivine'**Andesite**')

For simultaneous reaction process in the magma, $Mg_{2-p}Fe_pSiO_4$ and $[Ca_{1-p}Na_p][Si_{4-y}Al_y]\ O_8$ are the **'<u>Olivine series'</u>** and **<u>Feldspar series</u>** formulae that can be used to calculate all olivine and feldspar minerals from the melt $\mathfrak{W}$, where p is an integer and ranges from 0 to 2 in olivine crystals and 0 to 1 in feldspar crystals. At p equals to zero, 100% Forsterite(Fo) and anorthite(An) will crystallize simultaneously from the melt with chemical formulae of Mg_2SiO_4 and $Ca_1[Al_2Si_2]0_8$ and $ca_1[Al_2Si_2]O_8$ they are the **<u>sequences</u>** and **<u>initial values</u>** during crystallization of olivine and feldspar from the melt. Therefore, at p equals to 2 for olivine and 1 for feldspar, Forsterite (fo) and Anorthite(An) disappear and 100% of pure Fayalite (Fa) and Albite(Ab) will crystallize simultaneously with chemical formulae of $Fe_2SiO_{4(s)}$ and $Na_1[AlSi_3]O_8$ and they are the **<u>subsequences</u>** and **<u>modified values</u>** during crystallization of olivine and feldspar from the melt. Therefore forsterite, anorthite, fayalite, and albite are the end members of olivine and feldspar in the mafic olivine and feldspar series and they set in matrices to form rocks of equivalent compositions.

4.3.2 Metrical Matrix for Mafic Pyroxene and Feldspar: $[\beta_{2-p}\alpha_p](Z_0)_{\bar{p}}$

Pyroxene and feldspar in this category of Matrix series are called primary Pyroxene and feldspar in the mafic and felsic magma complexes because they are the first derivatives of Pyroxene and feldspar from the magma $\mathfrak{W}$ and these require simple substitutions of ions of the same charge and similar size according to Goldschmidt.

At about 2000^0C and 1000^0C of the melt $\mathfrak{W}$, the first formed crystal of smaller independent tetrahedron unit of forsterite, (Mg_2SiO_4) that is, the "**olivine type mineral**" in which its $(SiO_4)^{-4}$ independent tetrahedron unit can be linked by the silica tetrahedron $(SiO_4)^{-4}$ by sharing the two corners of each tetrahedron with oxygen atoms in the silica rich melt $\mathfrak{W}$ and then polymerized to form, new crystal phase of much larger regular structural tetrahedron unit of Enstatite($Mg_2Si_2O_6$) called .a chain silicate usually single chain of an **"indefinite structure",** which is a "**pyroxene** type mineral and simultaneously **Bytownite** type of feldspar is formed.

Then set equations (10) and (15) in matrices form as shown in equations (28) below.

$$\begin{bmatrix} \mathrm{Ca_4Na_1}[Al_2\,Si_2]0_8 \\ \\ [Mg_2]\,Si_2O_6 \end{bmatrix} + \begin{bmatrix} \mathrm{Na_1}[AlSi_3]0_8 \\ \\ [Fe_2]\,Si_2O_6 \end{bmatrix} = \begin{bmatrix} (\mathrm{Ca_1Na_1})Al_3\,Si_5]0_{16} \\ \\ [MgFe\,]\,Si_2O_6 \end{bmatrix} \quad (28)$$

$$\begin{bmatrix} Bytownite \\ Enstatite \end{bmatrix} + \begin{bmatrix} Albite \\ Ferrosilite \end{bmatrix} = \begin{bmatrix} Anorthoclase \\ Pyroxene \end{bmatrix}$$

$[Basalt]+\ [Rhyolite] = [\ An\ desite]$

$[Basalt]+[Rhyolite]=[An\ desite]$

$[basic\ basalt]+[acidic\ Rhyolite]=[Intermediate]$ (29)

if pl>α, then then the intermediate rock is Andesite

$[Basalt]+[Decite]=An\ desite$ (30)

if pl <α, then the intermediate rock is Trachyte

$[Basalt]+[Ryolite]=Trachyte$ (31)

if pl=α , then the Intermediate rock is the Hybrid mogonite

$[Basalt]+[Rhyolite]=hybrid\ Monzonite$ (32)

Given that;

$$\varepsilon = [Mg + Fe]_2 Si_2O_6$$

$$\varepsilon_z = Mg_2Si_2O_6$$

$$\eta_z = Fe_2Si_2O_6$$

Adam-Bashforth (1883) method of difference equation to the solution of differential equation as the size (δ) of growing embryos (Z_0) approach the critical growth radius (δ_c) to form a stable crystal nuclei (Z) and Cauchy's sequence and subsequence are used to analyze the Pyroxene group, using the two end members of Pyroxene series as shown Table 5, below.

$Mg_2\,Si_2\,O_6$ = Initial value and the sequence

$$Fe_2\,Si_2\,O_6 = \text{Modified value and the subsequences}$$

Therefore, λ_{η} represents the roots of the equation $s[\beta^{2-p} \propto^{p}](Z_0)\overline{\delta}$ for Pyroxene minerals and $[X_{1-p}\gamma_p][\mathcal{Q}_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

Therefore;

$$\eta = 2 \text{ for Pyroxene and } \eta = 0.2 \text{ for Feldspar}$$

Then;

$$\lambda_2 = (2, 2) \text{ for Pyroxene and } \lambda_2 = (0.2, 1) \text{ for Feldspars}$$

Table 5. The two End members of Pyroxene group and Feldspar group using Adams-Bashfort (1883) and Cauchy sequence

Adams-Bashfort (1883)	**Initial value**		**Modified value**		**Root**	
lim $max\,\lvert\eta_z-\varepsilon_z\rvert=\eta$ $\eta\to 0, 0\le p\le n$	$\mathscr{E}_z = n$		$\eta_z = p$		λ_2	λ_2
Chemical formula	$Mg_2Si_2O_6$	$Ca_4Na_1[Al_2\,Si_2]0_8$	$Fe_2Si_2O_6$	$Na_1[AlSi_3]0_8$	2, 2	0.4, 1
Geologic name	Enstatite	Bytownite	Ferrosilite	Albite	2, 2	0.4, 1
Cauchy's sequence	Sequence	Sequence	Subsequence	Subsequence	First	first
Lyapunov (1992)	Stable	Stable	Stable	Stable	Neutral	Neutral

Classification of pyroxene under control thermodynamic condition

➢**Pyroxene Class** = 1[basic ***Pyroxene***] + 2[intermediate **Pyroxene**] + i[acidic ***Pyroxene***].
➢**Pyroxene Minerals** = basic '***Enstatite*** '+ i[intermedite '***Hypersthene***'] + acidic '***Ferrosilite'*** .
➢**Pyroxene rocks = (pyroxene'Basalt')+(pyroxene'Granite')+i[(pyroxene'Andesite')]**

For simultaneous reaction process in the magma, $Mg_{2-p}Fe_pSi_2O_6$ and $[Ca_{1-p}Na_p][Si_{4-y}Al_y]O_8$ are the **'Pyroxene series'** and **Feldspar series** formulae that can be used to calculate all Pyroxene and feldspar minerals from the melt $\mathfrak{W}$, where p is an integer and ranges from 0 to 2 in olivine and 0 to 1 in feldspar crystal. At pequals to zero for olivine crystal, 100% Enstatite(En_{100}) and at p equals to 0.2 for feldspar crystal, 80% anorthite(An_{80}) called Bytownite will crystallize simultaneously from the melt with chemical formulae of $Mg_2Si_2O_6$ and $Ca_4Na_1[Al_2Si_2]O_8$ and they are the **sequences** and **initial values** during crystallization of olivine and feldspar from the melt. Therefore, at p equals to 2 for olivine and 1 for feldspar, Enstatite(En) and Bytownite(By) disappear and 100% of pure Ferrosilite (Fe) and Albite(Ab) will crystallize simultaneously with chemical formulae of $Fe_2Si_2O_6$ and $Na_1[AlSi_3]O_8$ and they are the **subsequences** and **modified values** during crystallization of pyroxene and feldspar from the melt. Therefore Enstatite, Bytownite, ferrosilite, and albite are the end members of olivine and feldspar in the mafic pyroxene and feldspar series and they set in matrices to form rocks of equivalent compositions.

●Therefore olivine polymerizes to pyroxene under thermodynamic control as shown below;

$$\beta(Z_0)_{\bar{o}} \to \beta(Z_0)_{\bar{p}}$$

As, Lim $1000^{\circ}c$

$1890^{\circ}c$

● Also more basic olivine (β_n), changes to more acidic olivine **(**α_p**)**, by substitution reaction as shown below;

$$As\ \underset{\qquad\qquad Lim \qquad\quad Lim}{\mathfrak{W}(Z_0)_{\bar{p}} \to \beta(Z_0)_{\bar{p}} \to \propto Z_0)_{\bar{p}}}$$

● Polymerization and isomorphous reaction of minerals in the magma under thermodynamic control as shown below;

$$\begin{tikzcd}
W(z) \arrow[r, "polymerization"] \arrow[d, "Lim"', "Solid"] & \beta(z)_0 \arrow[r, "polymerization"] \arrow[d, "Solid"'] & \beta(z)_p\Delta G=0 \\
\Delta G<0 & \propto(z)_0 \arrow[r, "polymerization"] & \propto(z)_0\ \Delta G>0
\end{tikzcd}$$

This means that olivine complete its crystallization when iron rich in olivine is precipitated and crystallized as fayalite and its Gibbs free energy is minimum at that point.

All the pyroxenes crystallized in this case are in orthorhombic system. Pyroxene considered are;

1. Mg2S2O6
2. 2Mg FeS2O6
3. Fe2S2O6
4. [Mg>Fe]Si2O6
5. [Mg <Fe]Si2O6

Therefore with $(\beta_{2-p}\ \alpha_p)\ (Z_0)_{\bar{p}}$ = $Mg_{2-p}Fe_pSi_2O_6$, Enstatite and Ferrosilite are formulated with respect to Bowen's and Goldschmidt concepts and with these formulations Bowen's and Goldschmidt concepts are mathematically connected.

4.3.3 Metrical Matrix for Mafic Amphibole and Feldspar

Magma with water content at moderate to low temperature, silica rich and higher viscosity. Water content causes partial pressure to the magma at moderate to low temperature. This means that pressure affects the crystallization either delay or hasten of mineral from the melt.

Amphibole and feldspar in this category of Matrix series are called primary Amphibole and feldspar in the mafic and felsic magma complexes because they are the first derivatives of Amphibole and Feldspar from the magma $\mathfrak{M}$ and these require simple substitutions of ions of the same charge and similar size according to Goldschmidt (1937).

In amphiboles, the "**pyroxene type mineral**" in which its $(Si_2O_6)^{-4}$ chain tetrahedron unit can be linked by the silica tetrahedron $(Si_2O_6)^{-4}$ by sharing the corners of adjacent tetrahedron with oxygen atoms in the silica rich melt $\mathfrak{M}$ and then polymerized to form, much larger regular structural tetrahedron unit of Kupfferite($Mg_7Si_8O_{22}(OH)_2$, a two parallel chain called a double chain silicate of an **"indefinite structure",** which is a "**amphibole type mineral**" that has different form (structure) and similar chemistry (composition) with the second formed crystal of enstatite and simultaneously **Labradorite** type of feldspar is formed along with the Kupfferite amphibole type mineral.

Given that;

$(\beta+\alpha)_\eta Z_0 = [\beta_{\eta-p}\alpha_p]Z_0$

$\eta = 7$

$(\beta+\alpha)_7 Z_0 = [\beta_{7-p}\alpha_p]Z_0$

$[\beta_7+\alpha_7)Z_0 = [7,0]+[0,7]$ (33)

Then set equations (10) and (33) in matrices form as shown in equations (37) below.

$Si_2O_{6(s)} + Si_2O_{6(s)} \rightarrow Si_4O_{12(s)}$ (34)

$[\beta_{7-p}\alpha_p](Z_0)_{\bar{a}}, at[7,0] = [\beta_7](Z_0)_{\bar{a}}$ (35)

$[\beta_{2-p}\alpha_p](Z_0)_{\bar{a}}, at[0,7] = [\alpha_7](Z_0)_{\bar{a}}$ (36),

$$\begin{bmatrix} Ca_3Na_2[Al_2Si_2]0_8 \\ [Mg_7]Si_8O_{22}(OH)_2 \end{bmatrix} + \begin{bmatrix} Na_1[AlSi_3]0_8 \\ [Fe_2]Si_8O_{22}(OH)_2 \end{bmatrix} = \begin{bmatrix} (Ca_1Na_1)[Q_5\ \mathcal{N}_3]0_{16} \\ [MgFe]_7Si_{16}O_{44}(OH)_4 \end{bmatrix} \quad (37)$$

$$\begin{bmatrix} Labradorite \\ Kupfferite \end{bmatrix} + \begin{bmatrix} Albite \\ Grunerite \end{bmatrix} = \begin{bmatrix} Anorthoclase \\ Anthophyllite \end{bmatrix}$$

[*Basalt*]+[*Rhyolite*]=[Andesite]

[*basic basalt*]+[*acidic Rhyolite*]=[*Intermediate*] (38)

if $pl > \alpha$ *, then then the intermedite rock is Andesite*

[*Basalt*]+[*Dacite*] = Andesite(39)

if $pl < \alpha$ *, then then the intermedite rock is Trachyte*

[*Basalt*]+[*Rhyolite*]=*Trachyte*(40)

if $pl = \alpha$ *, then then the intermedite rock is the Hybrid mogonite*

$[Basalt]+[Rhyolite]=hybrid\ Monzonite$ (41)

Given that;

$$\varepsilon = [Mg + Fe]_7 Si_8 O_{22}(OH)_2$$

$$\varepsilon_z = Mg_7 Si_8 O_{22}(OH)_2$$

$$\eta_z = Fe_7 Si_8 O_{22}(OH)_2$$

Adam-Bashforth method of difference equation to the solution of differential equation as the size (δ) of growing embryos (Z_0) approach the critical growth radius (δ_c) to form a stable crystal nuclei (Z) and Cauchy's sequence and subsequence are used to analyze the Amphibole group, using the two end members of Amphibole series as shown Table 6, below.

$$Mg_7 Si_8 O_{22}(OH)_2 = \text{Initial value and the sequence}$$

$$Fe_7 Si_8 O_{22}(OH)_2 = \text{Modified value and the subsequence}$$

Therefore, λ_η represents the roots of the equation $[\beta^{7-p} \propto^{p}](Z_0)\overline{\delta}$ for Amphibole minerals and $[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

Therefore;

$$\eta = 7 \text{ for Amphibole and } \eta = 0.4 \text{ for Feldspar}$$

Then;

$$\Lambda_7 = (7, 7) \text{ for Amphibole and } \lambda_1 = (0.4, 1) \text{ for Feldspars}$$

Table 6. The two End members of Amphibole group and Feldspar group using Adams-Bashfort (1883) and Cauchy sequence

Adams-Bashfort (1883)	**Initial value**		**Modified value**		**Root**	
$\lim max \lvert\eta_z-\varepsilon_z\rvert=\eta$ $\eta\to 0, 0\le p\le n$	$\mathcal{E}_z = n$		$\eta_z = p$		λ_2	λ_2
Chemical formula	$Mg_7Si_8O_{22}(OH)_2$	$Ca_3Na_2[Al_2\ Si_2]0_8$	Fe_7 $Si_8O_{22}(OH)_2$	$Na_1[AlSi_3]0_8$	2, 2	0.6, 1
Geologic name	Kupfferite	Labradorite	Grunerite	Albite	2, 2	0.6, 1
Cauchy's sequence	Sequence	Sequence	subsequence	Subsequence	First	first
Lyapunov (1992)	Stable	Stable	Stable	Stable	Neutral	Neutral

Classification of amphibole under thermodynamic condition

➢**Amphibole class** = 1[basic ***Amphibole***] + [mafic ***Amphibole***] + i[acidic ***Amphibole***].
➢**Amphibole Minerals** = basic '***Kupfferite*** ' + basic '***Anthophyllite***+ i[intermedite '***Anthophyllite'-'Cummingtonite***'] + acidic' ***Cummingtonite***' + acidic '***Grunerit***e'
➢**Amphibole rocks** = +(**amphibole'Basalt**')+(**amphibole'Andesite**')+i(**amphibole'Granite**')]

For simultaneous reaction process in the magma, $[(Mg_{7-p}Fe_p)(Si_8O_{22}(OH)_2]$ and $\left[Ca_{1-p}Na_p\right][Si_{4-y}Al_y]O_8$ are the '**Amphibole series'** and **Feldspar series** formulae that can be used to calculate all amphibole and feldspar minerals from the melt $\mathfrak{M}$, where p is an integer and ranges from 0 to 7 in amphibole and 0 to 1 in feldspar crystal. At p equals to zero for amphibole crystal, 100% Kupfferite(Kup_{100}) and at p equals to 0.4 for feldspar crystal, 60% anorthite(An_{60}) called Labradorite will crystallize simultaneously from the melt with chemical formulae of Mg_2SiO_4 and $Ca_3Na_2[Al_2Si_2]O_8$ and they are the **sequences** and **initial values** during crystallization of olivine and feldspar from the melt. Therefore, at p equals to 2 for amphibole and 1 for feldspar, Kupfferite(Ku) and Labradorite(La) disappear and 100% of pure Grunerite (Gu) and Albite(Ab) will crystallize simultaneously with chemical formulae of $Fe_2\,\beta_{7-p}\,SiO_{4(s)}$ and $Na_1[AlSi_3]O_8$ and they are the **subsequences** and **modified values** during crystallization of olivine and feldspar from the melt. Therefore Kupfferite, Labradorite, Grunerite, and albite are the end members of amphibole and feldspar in the mafic amphibole and feldspar series and they set in matrices to form rocks of equivalent compositions.

Therefore with (α_p) $(Z_0)_{\overline{p}}$ = $[(Mg_{7-p}Fe_p)(Si_8O_{22}(OH)_2]$, Kupfferite and Grunerite are formulated with respect to Bowen's and Goldschmidt concepts and with this formulation Bowen's and Goldschmidt concepts are mathematically connected. Therefore Kupfferite, Anthophyllite, Cummingtonite and Grunerite are the primary amphiboles in the mafic amphibole series.

4.3.4 Metrical Matrix for Black Mica and Feldspar

Mica and feldspar in this category of Matrix series are called primary Mica and feldspar in the mafic and felsic magma complexes because they are the first derivatives of Mica and Feldspar from the magma $\mathfrak{W}$ and these require simple substitutions of ions of the same charge and similar size according to Goldschmidt.

The crystal of amphibole interacts with the magma, so that the **$Si_4O_{11}(OH)$** of amphibole links to cation is being hydrolyzed in the presence of excess water in the magma shown in equation (190) below.

$Si_4O_{11}(OH)(aq) + 0H_{(aq)} \longrightarrow Si_4O_{10}(OH)_2(aq)$ (42)

In equation (42) above, one atom of oxygen in amphibole radical is being replaced by one atom of hydroxide atom in **$Si_4O_{11}\,(OH)^{-7}$** to form **$Si_4O_{10}(OH)_2^{-6}$**.

$[\beta_{6-p}\alpha_p](Z_0)_{\overline{m}}, at\ [6,0] = [\beta_6](Z_0)_{\overline{m}}$

$[\beta_{2-p}\alpha_p](Z_0)_{\overline{m}}\ at\ [0,6] = [\alpha_6](Z_0)_{\overline{m}}$

$$\begin{bmatrix} Ca_2Na_3[Al_2Si_2]0_8 \\ [Mg_6]Si_8O_{20}(OH)_4 \end{bmatrix} + \begin{bmatrix} Na_1[AlSi_3]0_8 \\ [Fe_2]Si_8O_{22}(OH)_4 \end{bmatrix} = \begin{bmatrix} [(Ca_1Na_1)[Al_3Si_5]0_{16} \\ [MgFe]_7\,Si_{16}O_{44}(OH)_4 \end{bmatrix} \quad (43)$$

$$\begin{bmatrix} Andesite \\ Ph\log opite \end{bmatrix} + \begin{bmatrix} Albite \\ Lepidomelane \end{bmatrix} = \begin{bmatrix} Anorthoclase \\ Bioyite \end{bmatrix}$$

$[Basalt] + [Rhyolite] = [Andesite]$

$[basic\ basalt] + [acidic\ Rhyolite] = [Intermediate]$ (44)

if $pl > \alpha$ *, then then the intermedite rock is Andesite*

$[Basalt] + [Dacite]$ = Andesite (45)

if $pl < \alpha$ *, then then the intermedite rock is Trachyte*

$[Basalt] + [Rhyolite] = Trachyte$ (46)

if $pl = \alpha$ *, then then the intermedite rock is the Hybrid mogonite*

$[Basalt] + [Rhyolite] = hybrid\ Monzonite$ (47)

$$\varepsilon = [Mg + Fe]_6\,Si_8O_{20}(OH)_4$$

$$\varepsilon_z = Mg_6Si_8O_{20}(OH)_4$$

$$\eta_z = Fe_6Si_8O_{20}(OH)_4$$

Adam-Bashforth (1883) method of difference equation to the solution of differential equation as the size (δ) of growing embryos (Z_0) approach the critical growth radius (δ_c) to form a stable crystal nuclei (Z) and Cauchy's sequence and subsequence are used to analyze the Mica group, using the two end members of Mica series as shown Table 7, below

$$Mg_6Si_8O_{20}(OH)_4 = \text{Initial value and the sequence}$$

$$Fe_6Si_8O_{20}(OH)_4 = \text{Modified value and the subsequence}$$

Therefore, λ_η represents the roots of the equation $s\ [\beta^{6-p}\propto^p](Z_0)\overline{\delta}$ for Mica minerals and $[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

Therefore;

$$\eta = 6 \text{ for Mica and } \eta = 0.6 \text{ for Feldspar}$$

Then;

$$\Lambda_7 = (6, 6) \text{ for Mica and } \lambda_1 = (0.6, 1) \text{ for Feldspars}$$

Table 7. The two End members of Mica group and Feldspar group using Adams-Bashfort (1883) and Cauchy sequence

Adams-Bashfort (1883)	**Initial value**		**Modified value**		**Root**	
lim *max* $\lvert\eta_z-\varepsilon_z\rvert=\eta$ $\eta\to 0, 0\le p\le n$	$\mathcal{E}_z = n$		$\eta_z = p$		λ_2	λ_2
Chemical formula	$Mg_6Si_8O_{20}(OH)_4$	$Ca_2Na_3[Al_2\,Si_2]0_8$	Fe_6 $Si_8O_{20}(OH)_4$	$Na_1[AlSi_3]0_8$	2, 2	0.6, 1
Geologic name	Phlogopite	Andesine	Lepidomelane	Albite	2, 2	0.6, 1
Cauchy's sequence	Sequence	Sequence	Subsequence	Subsequence	First	first
Lyapunov (1992)	Stable	Stable	Stable	Stable	Neutral	Neutral

Classification of mafic mica under controlthermodynamic condition

➢**Mica class** = 1[basic Mica] + [intermediate Mica] + i[acidic Mica].

➢**Mica Minerals** = basic '***Phlogopite*** ' + basic '***biotite***+ i[intermedite '***Biotite***'] + acidic' Biotite + acidic **'Lepidomelane'**

➢**Mica rocks** = (mica'**Basalt**')+(mica'**Granite**')+i[(mica'**Andesite**')]

For simultaneous reaction process in the magma, **[(Mg$_{6-p}$Fe$_p$)(Si$_8$O$_{20}$(OH)$_4$]** and $\left[Ca_{1-p}Na_p\right][Si_{4-y}Al_y]O_8$ are the **'Mica series'** and **Feldspar series** formulae that can be used to calculate all Pyroxene and feldspar minerals from the melt $\mathfrak{M}$, where p is an integer and ranges from 0 to 7 in olivine and 0 to 1 in feldspar crystal. At p equals to zero for olivine crystal, 100% Enstatite(En$_{100}$) and at p equals to 0.6 for feldspar crystal, 40% anorthite(An$_{40}$) called Andesine will crystallize simultaneously from the melt with chemical formulae of **[(Mg$_6$)(Si$_8$O$_{20}$(OH)$_4$]** and $Ca_2Na_3[Al_2Si_2]O8$ and they are the **sequences** and **initial values** during crystallization of olivine and feldspar from the melt. Therefore, at p equals to 6 for amphibole and 1 for feldspar, Phlogopite(Ph) and Andesine(An) disappear and 100% of pure Lepidomelane (Lp) and Albite(Ab) will crystallize simultaneously with chemical formulae of **[(Fe$_6$)(Si$_8$O$_{20}$(OH)$_4$]** and $Na_1[AlSi_3]O_8$ and they are the **subsequences** and **modified values** during crystallization of olivine and feldspar from the melt. Therefore phlogopite, Andesine, Lepidomelane, and albite are the end members of amphibole and feldspar in the mafic amphibole and feldspar series and they set in matrices to form rocks of equivalent compositions.

Therefore with $\beta_{6-p}\alpha_p(Z_0)_{\bar{m}}$ = $[(Mg_{6-p}Fe_p)(Si_8O_{22}(OH)_2]$, , Phlogopite and Lepidomelane were formulated with respect to Bowen's and Goldschmidt concepts and with this formulation Bowen's and Goldschmidt concepts are mathematically connected.

Therefore Phlogopite, Biotite and Lepidomelane are the primary mica in the mafic mica series.

4.3.5 Metrical Matrix for white Mica and Feldspar

Mica and feldspar in this category of Matrix series are called secondary Mica and feldspar in the mafic and felsic magma complexes because they are the third derivatives of Mica and Feldspar from the magma $\mathfrak{M}$ and these require substitutions of ions of different charges and different sizes provided there is couple substitution in another lattice for electrical neutrality according to Goldschmidt norm.

For complex reactions

Where, $Si_8O_{20}(OH)_4$ is the Mica radical group for all corresponding mica series in the light silicate complex?

$$\begin{bmatrix} Ca_1Na_4[Al_2\,Si_2]0_8 \\ \\ Na\,[Al_2Si_4O_{10}(OH)_2\,] \end{bmatrix} + \begin{bmatrix} K_1[AlSi_3]0_8 \\ \\ K\,[Al_2Si_4O_{10}(OH)_2\,] \end{bmatrix} = \begin{bmatrix} [(K_1Na_1)(Si_6\,Al_2)]0_{16} \\ \\ (Na\,K)[Al_4(Al_2Si_6)O_{20}(OH)_4\,] \end{bmatrix} \quad (48)$$

$$\begin{bmatrix} Oligoclase \\ Paragonite \end{bmatrix} + \begin{bmatrix} Albite \\ Muscovite \end{bmatrix} = \begin{bmatrix} Perthite \\ white\ Mica \end{bmatrix}$$

$$[sodic-Granite] + [Potassic-Granite] = [leuco-Granite]$$

$$\varepsilon = [Ca + Na]Al_2Si_2O_8$$

$$\varepsilon_z = Na[Al_2Si_4O_{10}](OH)_2$$

$$\eta_z = K[Al_2Si_4O_{10}](OH)_2$$

Adam-Bashforth (1883) method of difference equation to the solution of differential equation as the size (δ) of growing embryos (Z_0) approach the critical growth radius (δ_c) to form a stable crystal nuclei (Z) and Cauchy's sequence and subsequence are used to analyze the **Plagioclase feldspar group**, using the two end members of Feldspar series as shown Table 8, below.

$$Na[Al_4Si_8O_{20}](OH)_4 = \text{Initial value and the sequence}$$

$$K[Al_4Si_8O_{20}](OH)_4 = \text{Modified value and the subsequence}$$

Therefore, λ_η represents the roots of the equation $s[\beta^{2-p} \propto^p](Z_0)\overline{\delta}$ for Mica minerals and $[X_{1-p}\gamma_p][\mathcal{Q}_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

Therefore;

$$\eta = 4 \text{ for Mica and } \eta = 0.8 \text{ for Feldspar}$$

Then;

$$\Lambda_7 = (4, 4) \text{ for Amphibole and } \lambda_1 = (0.8, 1) \text{ for Feldspars}$$

Table 8. The two End members of white Mica group and Feldspar group using Adams-Bashfort (1883) and Cauchy sequence

Adams-Bashfort (1883)	**Initial value**		**Modified value**		**Root**	
lim $max\ \lvert\eta_z - \varepsilon_z\rvert = \eta$ $\eta \to 0, 0 \le p \le n$	$\mathscr{E}_z = n$		$\eta_z = p$		λ_2	λ_2
Chemical formula	$Na\ [Al_2Si_4O_{10}(OH)_2$	$Ca_1Na_4[Al_2\ Si_2]0_8$	$K\ [Al_2Si_4O_{10}($	$Na_1[AlSi_3]0_8$	1,1	0.8, 1
Geologic name	Muscovite	Oligoclase	Pargonite	Albite	1, 1	0.8, 1
Cauchy's sequence	Sequence	Sequence	Subsequence	Subsequence	First	first
Lyapunov (1992)	Stable	Stable	Stable	Stable	Neutral	Neutral

Paragonite and Muscovite are secondary and corresponding micas in the white mica complex. Therefore $Na\left[Al_4Si_8O_{20}(OH)_4\right] - K\left[Al_4Si_8O_{20}(OH)_4\right]$ *is the* alkaline series rock type of silicate. The presence of certain percentage of this alkaline series in mica rock type gives the name of the rock type e.g alkaline mica Rhyolite and these were formulated with respect to Bowen's and Goldschmidt concepts and with these formulations Bowen's and Goldschmidt concepts are mathematically connected.

For simultaneous reaction process in the magma, **$[(Fe_{4-p}Al_p)(Si_8O_{20}(OH)_4]$** and $\left[Ca_{1-p}Na_p\right][Si_{4-y}Al_y]O_8$ are the white **'Mica series'** and **Feldspar series** formulae that can be used to calculate all Pyroxene and feldspar minerals from the melt $\mathfrak{W}$, where p is an integer and ranges from 0 to 7 in olivine and 0 to 1 in feldspar crystal. At p equals to zero for olivine crystal, 100% Enstatite(En_{100}) and at p equals to 0.8 for feldspar crystal, 20% anorthite(A_{n20}) called Andesine will crystallize simultaneously from the melt with chemical formulae of **$[Na(Al_4)(Si_8O_{20}(OH)_4]$** and $Ca_1Na_4[Al_2Si_2]O_8$ and they are the **sequences** and **initial values** during crystallization of olivine and feldspar from the melt. Therefore, at p equals to 6 for amphibole and 1 for feldspar, Paragonite(Pa) and Andesine(An) disappear and 100% of pure Muscovite (Mu) and Albite(Ab) will crystallize simultaneously with chemical formulae of **$[K(Al_4)(Si_8O_{20}(OH)_4]$** and $Na_1[AlSi_3]O_8$ and they are the **subsequences** and **modified values** during crystallization of olivine and feldspar from the melt. Therefore Paragonite, Oligoclase, Muscovite, and albite are the end members of amphibole and feldspar in the mafic amphibole and feldspar series and they set in matrices to form rocks of equivalent compositions.

Therefore with $(\beta_{4-p}\alpha_p 0(Z_0)_{\overline{m}} = [(Fe_{4-p}Al_p)(Si_8O_{20}(OH)_4]$, , Paragonite and Muscovite were formulated with respect to Bowen's and Goldschmidt concepts and with this formulation Bowen's and Goldschmidt concepts are mathematically connected.

4.3.6 Metrical Matrix for Alkaline Minerals

Feldspars in this category of Matrix series are called primary feldspar in the felsic magma complexes because they are the first derivatives of Feldspar from the magma $\mathfrak{W}$ and this requires substitution of ions of different charges and different sizes provided there is couple substitution in another lattice for electrical neutrality .according to Goldschmidt.

If alkali earth metal is depleted from the melt $\mathfrak{W}(Z_0)$, *then*

$$\gamma_m[Q+\mathcal{N}]_{x+y}+[x]_p(Q+\mathcal{N})_{x+y}=(Q+\mathcal{N})_{x+y}\mathcal{W}_i$$

$$As,\ \lim\ \left\{Y_m\ [Q_{x-y}\mathcal{N}_y]\right\}\mathcal{W}_{i\to 0}$$

$$X_p(Q+\mathcal{N})_{x+y}\mathcal{W}_i=X_p\left[Q+\mathcal{N}\right]_{x+y}\mathcal{W}_i$$

Such that;

$$X_p\left[Q_{x-y}\mathcal{N}_y\right]\mathcal{W}_i=0\ (49)$$

As the alkali earth metals are depleted from the melt, $\mathfrak{W}(Z_0)$ more alkali metals of the same size and charge are being precipitated from the remaining melt, $\mathfrak{W}(Z_0)$ so that they can be interchangeable during solid solution. At a high temperature, alkali metals of the same charge can coexist together at the ratio of at least (90:10) or (64:36). The ratio in which they coexist is called Anorthoclase in felsic silicate rock and is shown in equation (49) above, below.

At a low temperature, the alkali metals exist in separate phases, and they become mirror image of the other, therefore one exceeds one another.

Therefore, $X_p[Q_{x-y}\mathcal{N}_y]\mathcal{W}_j=\mathcal{K}[X_p([Q_{x-y}\mathcal{N}_y)\mathcal{W}_j$

$$\mathcal{K}^*(X_p([Q_{x-y}\mathcal{N}_y)\mathcal{W}_j]=0$$

$$[\mathcal{K}^*X_p]^*[Q_{x-y}\mathcal{N}_y]\mathcal{W}_j=0$$

$$[\mathcal{K}X_y]^*[Q_{x-y}\mathcal{N}_y]\mathcal{W}_j=[\mathcal{K}+X]_{y+c}[Q_{x-y}]\mathcal{W}_j$$

$$[\mathcal{K}+X]_{p+c}[Q+\mathcal{N}]_{x+y}\mathcal{W}_j=[X_{p-c}\mathcal{K}_c][Q_{x-y}\mathcal{N}_y]\mathcal{W}_j\ (50)$$

$$[X_{p-c}\mathcal{K}_c][Q_{x-y}\mathcal{N}_y]\mathcal{W}_j=(NaK)Al_2\ Si_2O_8$$

$\therefore\ [X_{p-c}\mathcal{K}_c][Q_{x-y}\mathcal{N}_y]\mathcal{W}_j=$ Anorthoclase

$[\mathrm{X}_p:\mathcal{K}_c]=[90:10]$ or $[64:36]$

If $[X_{p-c}\mathcal{K}_c]\mathcal{W}_j=[X_p+\mathcal{K}_c]=[X+\mathcal{K}]_{p+c}$

$$[X_{p-c}\mathcal{K}_c][Q_{x-y}\mathcal{N}_y]\mathcal{W}_j=[X+\mathcal{K}]_{p+c}[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$$

$$[X+\mathcal{K}]_{p+c}[Q+\mathcal{N}]_{x+y}\mathcal{W}_i=0$$

Also,

$$[X+\mathcal{K}]_{p+c}=X_p+\mathcal{K}_c$$

$$[Q+\mathcal{N}]_{x+y}=Q_x+\mathcal{N}_y, then$$

$[X_p+\mathcal{K}_c][Q+\mathcal{N}_y]_{x+y}\mathcal{W}_j=X_p[Q+\mathcal{N}]_{x+y}+\mathcal{K}_c[Q+\mathcal{N}_y]_{x+y};so$ that

$X_p[Q+\mathcal{N}]_{x+y}+\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j=$ ***ALKALINE SERIES (ANORTHOCLASE***

$X_p[Q+\mathcal{N}]_{x+y}+\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j=$ $NaAlSi_3O_8$+$iKALSi_3O_8$

$$\underbrace{X_p[Q+\mathcal{N}]_{x+y}+\mathcal{K}_c[Q+\mathcal{N}]_{x+y}=0}_{\textit{Anorthoclase}\ \text{series}}\ (51)$$

$NaAlSi_3O_8$ + $iKAlSi_3O_8$ = ALKALINE FELDSPAR

Then equation (51) above, is called Anorthoclase series.

a. Perthi te-antiperthite series

$$X_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j+\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j=0$$

Then,

$X_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = -\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ (52)

$X_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = \mathrm{NaAlSi_3O_8}$

$-\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = K\mathrm{AlSi_3O_8}$

$NaAlSi_3O_8$ = Albite

$KAlSi_3O_8$ = Orthoclase

$$\underbrace{X_y[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = i\mathcal{K}_c([Q+\mathcal{N}]_{x+y}\mathcal{W}_j}_{Perthite} \quad (53)$$

$$\underbrace{NaAlSi_3O_8 = iKAlSi_3O_8}_{Perthite}$$

Therefore, equation (53) is called Perthite, because $X_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ *exceeds* $i\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ *in the felsic silicate rock*

If $\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = -X_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ then;

$$\underbrace{\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = iX_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j}_{Antiperthite} \quad (54)$$

$$\underbrace{KAlSi_3O_8 = iNaAlSi_3O_8}_{Antiperthite}$$

Therefore, equation (232) is called Antiperthite, because $\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ *exceeds* $\mathrm{i}X_y[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ *in the felsic silicate rock and* is expressed mathematically in the above equation.

Anorthite, Albite and Orthoclase are secondary and corresponding feldspars in the felsic complex and Therefore with $X_p[Q+\mathcal{N}]_{x+y}\mathcal{W}_j = -\mathcal{K}_c[Q+\mathcal{N}]_{x+y}\mathcal{W}_j$ Albite and Orthoclase were formulated with respect to Bowen's and Goldschmidt concepts hence, with these formulations Bowen's and Goldschmidt concepts are mathematically connected.

Where, $Al_2Si_2O_8$ is the Feldspar radical group for all corresponding Feldspar series in the light silicate complex?

$$\begin{bmatrix} \mathrm{K_1}[AlSi_3]0_8 \\ \\ Si\,O_2 \end{bmatrix} + \begin{bmatrix} \mathrm{Na_1}[AlSi_3]0_8 \\ \\ Si\,O_2 \end{bmatrix} = \begin{bmatrix} [(\mathrm{K_1Na_1})(Si_6\,Al_2)]0_{16} \\ \\ Si\,O_2 \end{bmatrix} \quad (55)$$

$$\begin{bmatrix} Orthoclase \\ \alpha\text{−}Quartz \end{bmatrix} + \begin{bmatrix} Albite \\ \beta\text{−}Quartz \end{bmatrix} = \begin{bmatrix} Perthite \\ Quartz \end{bmatrix}$$

$$[sodic\text{−}Granite] + [Potassic\text{−}Granite] = [\,Granite]$$

Given that;

$$\varepsilon = [Ca+Na]Al_2Si_2O_8$$

$$\varepsilon_z = K_1[AlSi_3]0_8$$

$$\eta_z = Na_1[AlSi_3]0_8$$

Adam-Bashforth (1883) method of difference equation to the solution of differential equation as the size (δ) of growing embryos (Z_0) approach the critical growth radius (δ_c) to form a stable crystal nuclei (Z) and Cauchy's sequence and subsequence are used to analyze the **Plagioclase feldspar group**, using the two end members of Feldspar series as shown Table 9, below.

$K_1[AlSi_3]0_8$ = Initial value and the sequence

$Na_1[AlSi_3]0_8$ = Modified value and the subsequence

Therefore, λ_η represents the roots of the equation $[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

$[X_{1-p}\gamma_p][Q_{4-y}\mathcal{N}_y]\mathcal{W}_i$ for Feldspar minerals

Therefore;

$\eta = 4$ for Mica and $\eta = 0.8$ for Feldspar

Then;

$\lambda_1 = (1, 1)$ for Feldspars

Table 9. The two End members of Feldspar group using Adams-Bashfort (1883) and Cauchy sequence

Adams-Bashfort (1883)	**Initial value**		**Modified value**		**Root**	
lim $max\ \lvert\eta_z - \varepsilon_z\rvert = \eta$ $\eta \to 0, 0 \le p \le n$	$\mathscr{E}_z = n$		$\eta_z = p$		λ_2	λ_2
Chemical formula	$-K\,[Al\,Si_3O_8\ ]$	SiO_2	$Na\,[Al\,Si_3O_8\ ]$	SiO_2	1, 1	0
Geologic name	Orthoclase	α -Quartz	Albite	β -Quartz	1, 1	0
Cauchy's sequence	Sequence	Sequence	Subsequence	Subsequence	First	first
Lyapunov (1992)	Stable	Stable	Stable	Stable	Neutral	Neutral

Orthoclase and Albite are primary and corresponding feldspars in the feldspar c omplex. Therefore $Na\,[Al\,Si_3O_8\ ] - K\,[Al\,Si_3O_8\ ]$ *is the* alkaline series rock type of silicate. The presence of certain percentage of this alkaline series in mica rock type gives the name of the rock type e.g alkaline mica Rhyolite and these were formulated with respect to Bowen's and Goldschmidt concepts and with these formulations Bowen's and Goldschmidt concepts are ma thematically connected.

For simultaneous reaction process in the magma, $\left[Ca_{\ 1-p}Na_p\right]\ [Si_{4-y}Al_y]\ O_8$ is the **Feldspar series** formulae that can be used to calculate all feldspar minerals from the melt $\mathfrak{W}$, where p is an integer and ranges from 0 to 1 in feldspar crystal. At p equals to 1 for feldspar crystal, orthoclase and 0% anorthite(A_{n0}) called Albite will crystallize simultaneously from the melt with chemical formulae of **$[K(Al)(Si_3O_8]$** and $Na_1[AlSi_3]O_8$ and they are the **sequences** and **initial values** during crystallization of feldspar from the melt. Therefore Orthoclase and Albite are the end members of Alkali feldspar in the feldspar series and they set in matrices to form rocks of equivalent compositions.

Therefore with, $\left[Ca_{\ 1-p}Na_p\right]\left[Si_{4-y}Al_y\right]O_8$, Orthoclase and Albite were formulated with respect to Bowen's and Goldschmidt concepts and with this formulation Bowen's and Goldschmidt concepts are mathematically connected.

4.4 Application of set Theory to Rock Forming Minerals in Magma

The theory of sets is an important tool in modern Mathematics. The study of sets has assumed a central role in every branch of Mathematics today. Set theory can be applied in geology to study the distribution of trace and or REE in rock after extensive geochemical analysis.

It is more precise to use set theory to probe or discriminate and delineate the distribution of trace elements in basaltic liquid, rather than using matrix method throughout the time of crystallization of magma after geochemical analysis using XRF.

4.4.1 Mathematical Analysis of Trace Elements in Mafic and Felsic Rocks.

The distribution of trace elements in mafic and felsic rock are greatly influenced by substitution of major element for trace elements throughout the time of crystallization of magma under control condition of temperature, pressure, composition of the liquid and solid, etc., according to Goldschmidt..

These substitutions depend on compactibility and incompatibility of trace elements, the ability of trace elements to partition either in solid phase or in liquid phase.

A. Compactible Trace Element

In an equilibrium system composed of the crystal, and coexisting silicate melt compactible trace elements are those that partition in solid phase. For trace elements to substitute for major elements during crystallization of silicate melt. You considered that;

I. Their atoms must be approximately the same size e.g. substitution of Ni for Mg and Fe in ferromagnesian minerals.
II. Their atoms must be of similar charges e.g. Rb^+ replaces k^+.

B. Incompatible Trace Element

In an equilibrium system composed of the crystal, and coexisting silicate melt, incompatible trace elements are partitioned into the silicate melt. Incompatible trace elements can be subdivided based on its ionic potential (i), which is express as the ratio of charge in valence unit (z) to its radius in Angstrom unit(r) as shown in equation (56). Therefore;

I = Z/R (56)

I = ionic potential

Z = charge

R = radius

Therefor elements with Z/R > 2 are classed as high field strength elements. They are generally incompatible because of the difficulty in achieving charge balance when these ions with Z =+3, to +6 substitute for major element cations with Z =+2. Elements with Z/R < 2 are classed as large ion lithophile elements, and hence incompatible elements. They are also low field strength elements.

C. Distribution of Trace Elements According To Partition Coefficient

The distribution of trace elements in mafic and felsic rocks depends on whether the trace elements are partition into the solid phase or into the silicate melt. It is assumed that at equilibrium, the ratio of the concentration of trace element in solid phase (Cs) to its concentration in the liquid (Cl) is constant as shown in equation (57), below.

D =Cs/Cl (57)

Where D = partition coefficient.

When D > 1, a particular trace element is partitioned into the solid phase, and when D< 1, a particular element is partitioned into liquid phase provided that equilibrium is prevailed.

Analogy

The mathematical equations above can be represented using set Notation as shown in Figure 2, below.

Given that;

$$K_n=a, b, c, d, e, f, g, h,$$

Such that

$$a, b, g=1$$

$$c, d <1$$

$$\therefore b, e, f\ \textit{are partitioned along the solid phase}\ (C_s),$$

c, d, are partioned along the Liquid phase (C_l)

a b g, are partitioned along the solid phase (C_s) and the Liquid phase (C_l)

$$\therefore C_s=b, e, f$$

$$C_l=c, d$$

$$\frac{C_s}{CL}=a, b, g= C_s\ n\ C_L$$

$$C_s\ u\ C_L=a, b, c, d, e, f, g, h$$

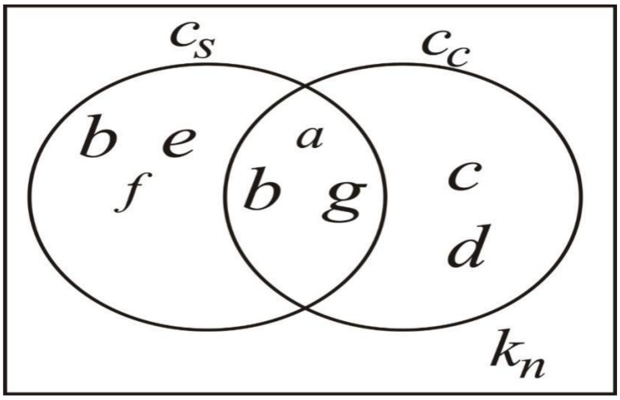


Figure 2. Set notation

4.5 The Distribution of Rare Earth Elements (ree) in Basaltic Liquid and its Analysis Using set Notation Method

The Table 10 below shows the distribution of Rare Earth elements in amphibole, pyroxene, magnetite, and plagioclase.

Table 10. The distribution of Rare Earth elements (REE) in Basaltic Liquid

REE	Amphibole	Pyroxene	Magnetite	Plagioclase
La	0.54	0.33	0.29	0.16
Ce	0.98	0.56	0.35	0.12
Na	2.1	1.1	0.0	0.07
Sm	2.99	1.76	0.55	0.061
Eu	2.88	1.55	0.55	0.79
Dy	4.3	2.3	0.0	0.0
Yb	2.29	1.56	0.26	0.026
Lu	2.3	1.9	0.6	0.0

Using the set theory and Venn diagram, probe or discriminate Rare Earth elements (REE) in Basaltic liquid in Table 10 above, given the fact that $K_d > 1$, where K_d, = partition coefficient.

$$K_d = [K_d \text{ minerd } [(1 \text{ m the rock})]+[\mathfrak{M}_{inz}]+[\mathfrak{M}_{in}\,\mathfrak{Z}]$$

20%py, 10% mag, 40%plag, 30% Am

$K_d=(0.2x1.76)+(0.1x0.53)+(0.4x0.061)+0.3x2.99=1.32$

Using set Notation,

$Am \cup Py \cup Pl$ = La, Ce, Nd, Sm, Eu, Dy, Yb, Lu

1. $(Am)\cap(Py) = (0.2x1.76) + 0.3x2.99$
$(Am)\cap(Py) = 1.249$
$(Am)\cap(Py) > 1$
2. (Am) (Pl) = Nd, Sm, Eu, Dy, Yb, Lu
$(Am)\cap(Py) = (0.4x0.061)+(0.2x1.76)$
$(Am)\cap(Py) = 0.3764$
$(Am)\cap(Py) < 1$
3. (Pl) (Py) = Nd, Sm, Eu, Dy, Yb, Lu
$(Am)\cap(Py) = (0.2x1.76)+0.3x2.99$
$(Am)\cap(Py) = 1.249$
$(Am)\cap(Py) > 1$

The Venn diagram is shown below;

$(Am)\cap(Py)>1$

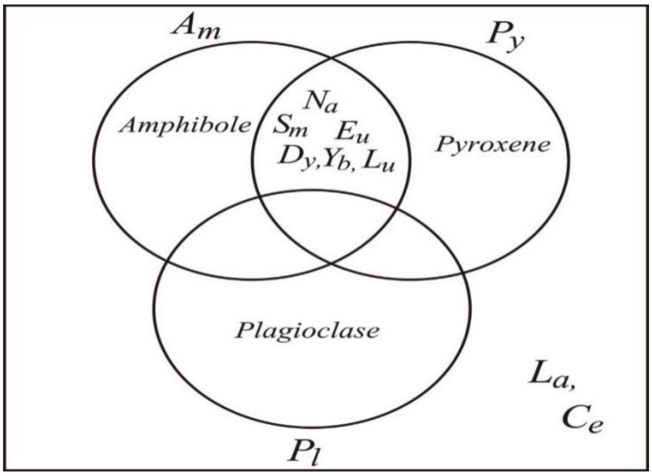


Figure 3. The Venn diagram showing the distribution of Rare Earth elements (REE) in Basaltic Liquid

From the Venn diagram above, Nd, Sm, Eu, Dy, Yb, Lu, are partitioned along the solid phases of Amphibole and pyrxene at the end of crystallization.

La, Ce, are partitioned along the fluid phase at the end of crystallization. They are high field strength elements, and cannot enter into the silicate structure, and remain in the fluid phase within their stability field to form gem minerals & ore deposit

4.6 Typical Partition Coefficient of Trace Elements between Crystals and Liquid and its Analysis using set Notation Method

Table 11. Typical Partition Coefficient of Trace Elements between Crystals and Liquid

	Trace Elementl		**Trace elements in Basaltic liquid: from J.G., Arth, 1976, Jour.Res.U.S. Geol. Surv. ,4: 41-47.**							**Trace elements in Basalt of KASSA from the researcher.**				
		Olivine	**Pyroxene**	Pyroxene	Amphibole	Mica	Plagioclase	Spinel	Garnet	Converted Data in KASSA x100				
S/N		*1*	2	3	*4*	*5*	6	*7*	*8*	*KIA1*	*KIA2*	*KIA3*	*KIA4*	
1	Ni	4-10	8.3	2.5	6.0	7.6	0.05	5.0	0.5	1.54		0.17	1.68	
2	Cr	0.2	2.0	11.5	5.2	7.0	0.06	10.0	2.0	2.26		0.07	1.72	2.15
3	Co	3.9	2.4	1.0	6.5	1.1	0.05	2.0	3.2	0.46		0.35	0.49	0.50
4	Sc	0.2	1.2	2.7	3.5	3.0	0.03	2.0	3.4					
5	Sr	0.01	0.03	0.11	0.6	0.1	2.1	<0.1	<0.1	4.45		9.59	6.63	7.81
6	Ba	0.02	0.05	0.02	0.4	<0.1	0.38	<0.1	<0.1	4.23		8.77	8.89	5.11
7	Rb	0.02	0.006	0.03	0.4	2.0	0.09	<0.1	<0.1					
	Mn									1.17		1.35	1.43	1.35
	Zr									1.56		2.71	1.92	1.97
	V									1.69		1.93	1.69	1.67
	Zn									1.07		1.33	1.17	1.31

Using set theory to probe or discriminate trace elements in Basaltic liquid, given the fact that D > 1, where D, = partition coefficient, then, from Table 11 above, trace elements in;

Olivine (O_l) = Ni, Co
Pyroxene (P_y) = Ni, Cr, Co, Sc
Plagioclase (P_l) = Sr

Suppose that all the trace elements in olivine, pyroxene and plagioclase are included in **Basaltic** liquid, it is pertinent to know that, not all the trace elements in the same **Basaltic** liquid are included in essential olivine, pyroxene, and plagioclase. This is because, the concentration of trace elements in solid to liquid depends on its "**Partition Coefficient**" (D).

Given that, **Basaltic** liquid is a universal set of all included trace elements, $\cup$, then according to Prinz (1967)

U= {Ba, Sr, Ni, Cr, Ga. Li, V, Sc, Rb, Co, Cu}

Using set notation,

If O_l, P_y, P_l, Z X_n, are sets of included trace elements, then their union is the set of all included trace elements which belong to at least one of them and it is denoted by;

$O_l \bigcup P_y .. \cup X_n = \bigcup X_i$

If the union is infinite, then, O_l, P_y, P_l, Z, X_n is given by

$O_l \cup P_y \cup .. \cup X_n \cup . = \cup \ X_i$. (57)

$\cup \ X_i = \{z \in \cup \ \}$ or $\{O_l \in \cup \ \}$ or $\{P_y \in \cup \ \}$ or $\{P_l \in \cup \ \}$. (58)

$\bigcup X_i = \{Z\} \cup \{O_l \cup P_y\} \cup \{P_l\}$. (59)

$\bigcup X_i =$ Basaltic liquid

The intersection, $\cap$ of sets O_l, P_y, P_l, ZX_n, is the sets of all included trace elements in the Basaltic liquid which belong to every one of them and is denoted by

$O_l \cap P_y n . \cap X = \bigcap X_i$ (60)

If the sets are infinite that is,

$O_l \cap P_y n \cap Xn = \bigcap X_i$ (61)
Let the set of included trace element in olivine, pyroxene and pyroxene are represented using set notation below,

Given that,

❖$x \in O_l$, $x \in P_y$ and $y \in P_l$, but $z \notin \{x, y, z\}$, where x and y are set of trace elements in olivine, pyroxene and plagioclase and z is the remaining trace elements in the **basaltic** liquid, then

$O_l \cap P_y = \{x: x \in O_l, \text{ and } x \in P_y\}$

$\underline{\mathbf{O_l \cap P_y}} = \{\text{Ni, Co}\}$

$O_l \cap P_l = \{x: x \in O_l\} \text{ and } \{y: y \in P_l\}$

$\underline{\mathbf{O_l \cap P_l}} = \emptyset$
$P_y P_l = \{x: x \in P_y\} \text{ and } \{y: y \in P_l\}$

$\underline{\mathbf{P_y \cap P_l}} = \emptyset$

$O_l \cap P_y \cap P_l = [\{x: x \in O_l, x \in P_y\} \text{ and } \{y: y \in P_l\}]$

$\underline{\mathbf{O_l \cap P_y \cap P_l}} = \emptyset$
$O_l \cup P_y = \{x: x \in O_l, \text{ or } x \in P_y\}$

$\underline{\mathbf{O_l \cup P_y}} = \{\text{Ni, Cr, Co, Sc,}\}$

$O_l \cup P_l = \{x: x \in O_l\}, \text{ or } \{y: y \in P_l\}$

$\underline{\mathbf{O_l \cup P_l}} = \{\text{Ni, Co, Sr}\}$

$P_y \cup P_l = \{x: x \in P_y\} \text{ or } \{y: y \in P_l\}$

$\underline{\mathbf{P_y \cup P_l}} = \{\text{Ni, Co, Cr, Sc, Sr,}\}$

$O_l \cup P_y \cup P_l = [\{x: x \in O_l, \text{ or } x \in P_y\} \text{ or } \{y: y \in P_l\}]$

$\underline{\mathbf{O_l \cup P_y \cup P_l}} = \{\text{Ni, Co, Cr, Sc, Sr,}\}$

$\{Z\} \cup \{O_l \cup P_y \cup P_l\}' = [\{z: z \notin O_l, \text{ or } z \notin P_y \text{ or } z \notin P_l\}]$

$\underline{\mathbf{\{Z\} \cup \{O_l \cup P_y \cup P_l\}}} = \{\text{Ba, Ga, Li, V, Rb, Cu}\}$

Using a vein diagram in Figure4 below, to represent the above set notation, we have;

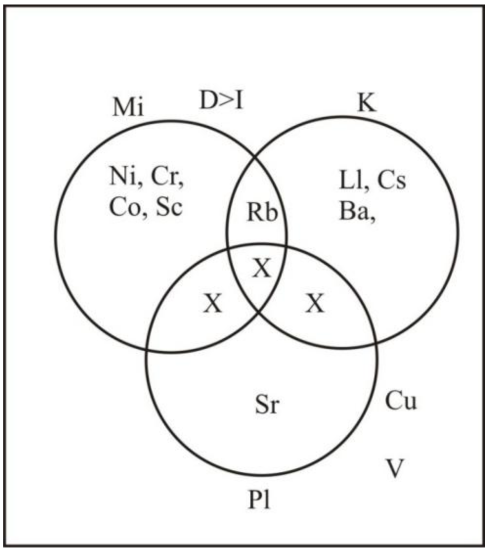


Figure 4. Vein diagram representing partition coefficient of trace element in leucogranite

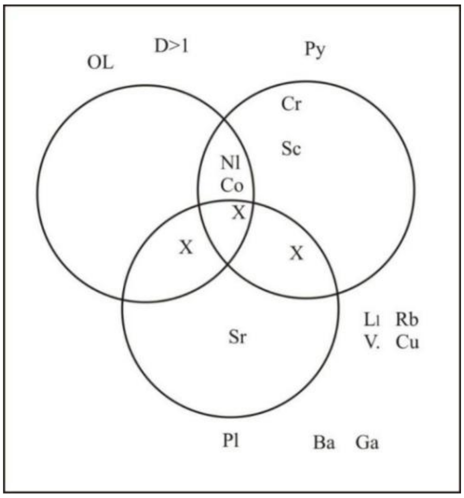
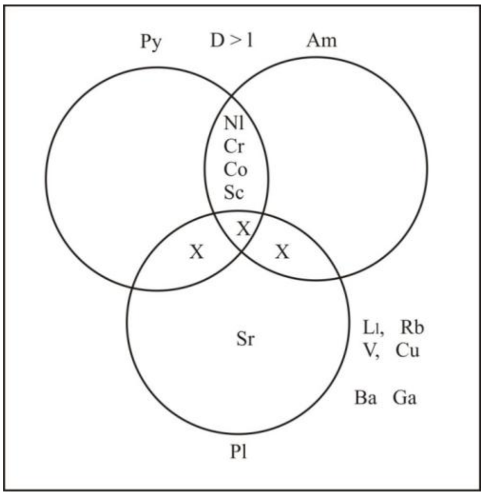


Figure 5. Vein diagram representing partition coefficient of trace element in Basalt



$Am = \{N_l, Cr, Co, Sc\}$

$Py \cap Am = \{N_l, Cr, Co, Sc\}$

$Am \cap Pl = Q$

$Py \cup Am \cup Pl = \{N_l, Cr, Co, Sc, Sr\}$

Figure 6. Vein diagram representing partition coefficient of trace element in intermediate rock

From Figure 6, above, that at a given equilibrium during crystallization of magma, those trace elements within the circles are partitioned in the solid phase and are compatible trace elements, while those outside the circles are partitioned into the liquid phase and are incompatible trace elements which are ready to be partitioned, if any of the conditions is altered such as drop in temperature. It is observed from the Figure 7, that olivine is a proper subset of pyroxene, with the union of plagioclase are mutually inclusive in the Basalt.

Therefore, $O_l\, P_y$ is read as "O_l is proper subset of P_y".

$O_l \subset P_y = O_l \cap P_y : O_l \cup P_y$

$\quad = \{x \in O_l, \text{ and } x \in P_y\}$

$O_l \cap P_y = O_l$

$O_l \cup P_y = P_y$

$\{O_l \cap P_y\} \cap \{P_l\} = [\{x: x \in O_l, x \in P_y\} \text{ and } \{y: y \in P_l\}]$

$\quad = \emptyset$, mutually exclusive.

$\{O_l \cup P_y\} \cup \{P_l\} = P_y \cup P_l$, mutually inclusive.

$\{O_l \cup P_y\} \cup \{P_l\} = [\{x: x \in O_l, \text{ or } x \in P_y\} \text{ or } \{y: y \in P_l\}]$

$\quad =$ Olivine Basalt

$P_{y\cup}P_l = [\{x: x \in P_y\} \text{ or } \{y: y \in P_l\}]$

$= \text{Tholeiite Basalt}$

Olivine Basalt → Tholeiite Basalt

Undersaturation → Oversaturation of Olivine.

Undersaturation of olivine called "Olivine Basalt" to oversaturation of olivine called "Tholeiite Basalt", means that there is enough silica, according to Kennedy (1933) and included trace elements to convert all of the olivine to pyroxene. Therefore, all the trace elements in olivine are included in pyroxene, but not all the trace elements in pyroxene are included in olivine using set notations and vein diagrams as shown Figures 2, 3, 4, 5and 6 above.

4.7 Bowen's and Goldschmidt Combined Models of Minerals and Rocks

Using Matrix and Set notation methods, this research work classifies minerals according to Goldschmidt and Bowen's concepts as shown in Figures 7, 8 and 9 below and minerals progress to the next minerals by polymerization reaction by addition of more silica to the already crystallized minerals in a more silica rich melt. This means that according to Bowen's, with drop in temperature of magma olivine progresses to pyroxene by adding silica to olivine mineral, pyroxene progresses to amphibole by addition of silica to pyroxene mineral, amphibole progresses to mica by addition of silica to amphibole crystal, and so on. After this, Bowen's postulated that the remaining melt at lower temperature is quenched to produce the last crystals such as orthoclase, muscovite and quartz. This progressive reaction is what Bowen's called ***discontinuous reaction series***, but for the purpose of this research, it is called ***polymerization reaction*** because of addition of silica to the already crystallized crystal under thermodynamic change.

Bowen's considers the reaction series of plagioclase feldspar as ***Continuous reaction series*** because of its continuous exchange of element for one another with drop in temperature. For one element to substitute for one another in the Goldschmidt space, it must follow Bowen's index number 1 to 7 as shown in Table 12 below, such that first Bowen's index number under thermodynamic change, must undergo ionic substitution provided that there is Diadochi in the first formed crystal in the Goldschmidt space.

The first Bowen's index number is ***Forsterite*** of olivine, with Bowen's Index number 1(one) in the Bowen's series. '***Forsterite*** 'under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form fayalite as the last end member of olivine in the olivine series. Both ***Forsterite***, wholly magnesium rich olivine and ***Fayalite***, wholly iron rich olivine are formed within the olivine series with Bowen's index number 1(one) as shown in Table 12, Figures, 7, 8, and 9, below. ***Forsterite*** of olivine is the first sequence of mineral to crystallize from the melt, by Cauchy's sequence and initial value by Adams-Bashforth method while Fayalite is subsequence by cauchy's sequence and modified value by Adams-Bashforth method. This means by Cauchy's definition of sequence, for all minerals in **Olivine group**, there exist, **Forsterite**, and **Fayalite**, in ***{Olivine group}***, such that is the first member and initial mineral of {**Olivine group**} and **Fayalite** is the last member and modified mineral of {**Olivine group**}.

The second Bowen's index number is ***Enstatite*** of pyroxene with Bowen's Index number 2(Two) in the Bowen's reaction series and the '***Enstatite***' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form ***Ferrosilte*** as the end member of pyroxene in the pyroxene series. Both ***Enstatite***, wholly magnesium rich pyroxene and ***Ferrosilite***, wholly iron rich pyroxene are formed within the pyroxene series with Bowen's index number 2(two) as shown in Table 12, Figures 7, 8 and 9 below. ***Enstatite*** of pyroxene is the second sequence of mineral to crystallize from the melt, by Cauchy's sequence and initial value by Adams-Bashforth method while Ferrosilite is the subsequence by cauchy's sequence and modified value by Adams-Bashforth method. This means by Cauchy's definition of sequence, for all minerals in **Pyroxene group**, there exist, **Enstatite** and **Ferrosilite**, in ***{Pyroxene group}***, such that is the first member and initial mineral of {**Pyroxene group**} and **Ferrosilite** is the last member and modified mineral of {**Pyroxene group**}.

The third Bowen's index number is '***Kupfferite***' of amphibole with Bowen's Index number 3(Three) in the Bowen's reaction series and the '***Kupfferite***' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form ***Grunerite*** as the end member of amphibole in the amphibole series. Both ***Kupfferite***, wholly magnesium rich amphibole and ***Grunerite***, wholly iron rich amphibole are formed within the amphibole series with Bowen's index number 3(three) as shown in Tables 12, Figures 7, 8, and 9 below. ***Kupfferite*** of amphibole is the third sequence of mineral to crystallize from the melt, by Cauchy's sequence and initial value by Adams-Bashforth method while Grunerite is the subsequence mineral to crystallize from the melt, by Cauchy's sequence and modified value by Adams-Bashforth method. This means by Cauchy's definition of sequence, for all minerals in **Amphibole group**, there exist, **Kupfferite**, and **Grunerite**, in ***{Amphibole group}***, such that is the first member and initial mineral of {**Amphibole group**} and **Grunerite** is the last member and modified mineral of {**Amphibole group**}.

The Fourth Bowen's index number is '***Phlogopite***' of mica with Bowen's Index number 4(Four) in the Bowen's reaction series and the '***Phlogopite***' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form ***Lepidomelane*** as the end member of mica in the mica series. Both ***Phlogopite***, wholly magnesium rich mica and ***Lepidomelane***, wholly iron rich mica are within the mica series with Bowen's index number 4(four) as shown in Table 12, Figures 7, 8 and 9 below. ***Phlogopite*** of mica is the fourth sequence of mineral to crystallize from the melt, by Cauchy's sequence and initial value by Adams-Bashforth method while Lepidomelane is the subsequence mineral to crystallize from the melt, by Cauchy's sequence and modified value by Adams-Bashforth method. This means by Cauchy's definition of sequence, for all minerals in **Mica group**, there exist, **Phlogopite**, and **Lepidomelane**, in ***{Mica group}***, such that is the first member and initial mineral of {**Mica group}** and **Lepidomelane** is the last member and modified mineral of {**Mica group}**.

Number 5 in the series according to Bowen's is ***Orthoclase***, a potassium rich alkali feldspar and has an Index number of 5. Sometimes sodium can substitute for potassium in orthoclase to form the end member, ***albite***, a sodium rich alkali feldspar in the alkali series as shown in Table 12, Figures 7, 8 and 9 below. Both the potassium rich orthoclase and sodium rich Albite are formed within the alkali feldspar series and they exist at high temperature as ***Perthite – Antiperthite*** mixed feldspar and have separate phase at lower temperature as perthite when orthoclase exceeds albite and ***Antiperthite*** when albite exceeds orthoclase. This process is called ***EXSOLUTION*** in rock forming minerals from the alkali melt α_m **.** ***Orthoclase*** of alkali feldspar is the fifth sequence of mineral to crystallize from the melt, by Cauchy's sequence and initial value by Adams-Bashforth method while **Albite** is the subsequence mineral to crystallize from the melt, by Cauchy's sequence and modified value by Adams-Bashforth method. This means by Cauchy's definition of sequence, for all minerals in **alkali feldspar**, there exist, **orthoclase**, and **Albite**, in ***{Alkali feldspar group}***, such that is the first member and initial mineral of {**Alkali feldspar group}** and **Albite** is the last member and modified mineral of {**Alkali feldspar group}**.

The sixth Bowen's index number is '***Muscovite***' of white mica with Bowen's Index number of 6(six) in the Bowen's reaction series and the '***Muscovite***' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form ***Paragonite*** as the end member of mica in the white mica series. Both ***Muscovite***, wholly potassium rich white mica and ***Paragonite***, wholly sodium rich white mica are formed within the white mica series with Bowen's index number 6(six) as shown in Table 12, Figures 7, 8 and 9 below. The process in which minerals formed by substitution of one element for anther using Goldschmidt concept is called ***Isomorphous*** series in this research. ***Muscovite*** of white mica is the sixth sequence of mineral to crystallize from the melt, by Cauchy's sequence and initial value by Adams-Bashforth method while Grunerite is the subsequence mineral to crystallize from the melt, by Cauchy's sequence and modified value by Adams-Bashforth method. This means by Cauchy's definition of sequence, for all minerals in **white mica group**, there exist, **Muscovite**, and **Paragonite**, in ***{White mica group}***, such that is the first member and initial mineral of {**White mica group}** and **Paragonite** is the last member and modified mineral of {**White mica group}**.

Table 12. Order of Crystallization of Minerals using Goldschmidt Concept

Temp	**Felsic**	**Mafic**	Olivine	Pyroxene	Amphibole	Black mica	Feldspar	White mica	Quartz
2000	Anorthite	1	Forsterite						
1000	Bytownite	2	Mgnesio-chrysolite	Enstatite					
900	Labradorite	3	Chysolite	Mnesio-hypersthene	Kupfferite				
800	Andesine	4	Hyalosiderite	Hypersthene	Magnesio-Anthophyllite	Phlogopite			
700	Oligoclase	5	Hortonolite	Ferro-hypersthene	Anthophyllite	Biotite	Orthoclase		
600	Albite	6	Ferro-hortonolite	Eulite	cummingtonite	Ferro-biotite	Perthite	muscovite	
500	Quartz	7	Fayalite	Ferrosilite	Grunerite	Lepidomelane	Albite	Paragonite	Quartz

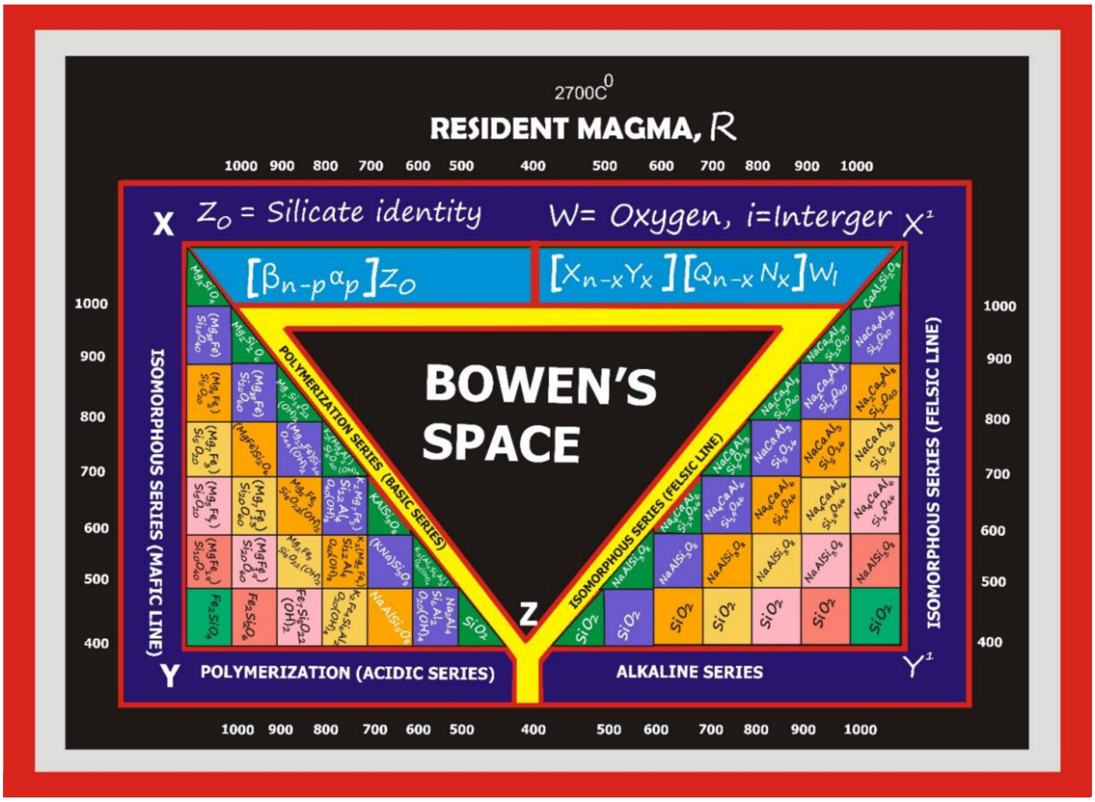


Figure 7. Mathematical Computations of Minerals in Bowen's and Goldschmidt Combined Model

❖XYZ = Mafic Compositions

❖X^1Y^1Z = Felsic Compositions

Where;

1.XYZ And X^1Y^1Z = Goldschmidt space

2.XX^1Z = Bowen's space.

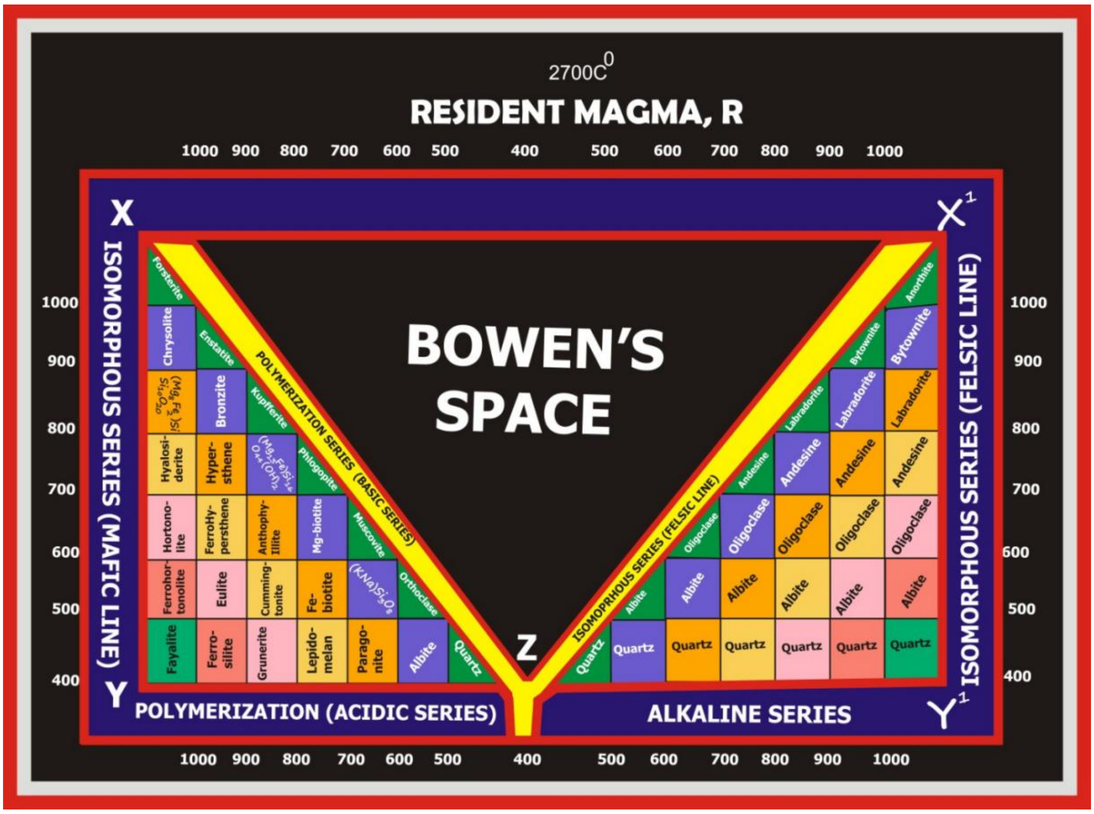


Figure 8. Mathematical Computations of Minerals in Bowen's and Goldschmidt Combined Model

❖XYZ = Mafic Minerals

❖X^1Y^1Z = Felsic Minerals

Where;

1. XYZ And X^1Y^1Z = Goldschmidt space
2. XX^1Z = Bowen's space.

4.8 Bowen's Index Number of Rock Forming Minerals and its Relationship with Goldschmidt Concept

Bowen's index number of rock forming minerals is the identity of minerals in Rocks under microscopic studies either plane or cross polarized light as is used in this research. The identity of minerals in rocks literally means identification of minerals in rocks using microscope. Analogically, if the mineral being observed under microscope is olivine, therefore the identity of minerals being observed in rock under microscope is actually the '**olivine miner*al***' and the identification number called Bowen's index is 1 (one) as shown Table 13 and Figure 9 below. This means in Bowen' reaction series olivine is 1 and the first mineral to crystallize from the melt. Pyroxene has identification number of 2, amphibole 3 and that of mica is 4 as shown in Table 13 Figure 9 below.

After identification number, the next is mineralogical name of the minerals and how it mathematically transforms from one form to the other within their own structural identity under thermodynamic change. Analogically, the first olivine mineral that forms is enriched in magnesium and the mineralogical name is forsterite, and is '**a sequence'** according to Cauchy sequence and '**initial value**', according to Adams-Bashforth method, therefore in mathematical relation to Goldschmidt concept as shown in Figure (9), iron substitutes for magnesium in forsterite crystal, at least about 100% to produce Fayalite of the same structure as the forsterite or which has the same structural identity as the forsterite. This continues as enstatite changes to Ferrosilite of the same structural identity, kupfferite changes to Grunerite of the same structural identity, and phlogopite changes to lepidomelane of the same structural identity as shown Table 13, Figure 9 below. Therefore the sequence and subsequence by Cauchy's definition of sequence and as well as initial and modified value by Adams – bashforth method of this reaction process mathematically connects the relationship between **Bowen's concept** and the **Goldschmidt concept** in this research. Hence this gives the complete explanation of rock forming minerals throughout the time of crystallization under thermodynamic change to some extent

Table 13. Sequence and Subsequence of Minerals by Cauchy sequence using Bowen's and Goldschmidt concepts

Bowen's Index Number	Mineral observed under thin section	Sequence/Initial value	Subsequence/Modified value
1	Oli vene	Forsterite	Fayalite
2	Pyroxene	Enstatite	Ferrosilite
3	Amphibole	Kupfferite	Grunerite
4	Mica	Phlogopite	Lepidomelane
5	Mica	Muscovite	Paragonite
6	Alkaline Feldspar	Orthoclase	Albite
7	Plagioclase feldspar	Anorthite	Albite

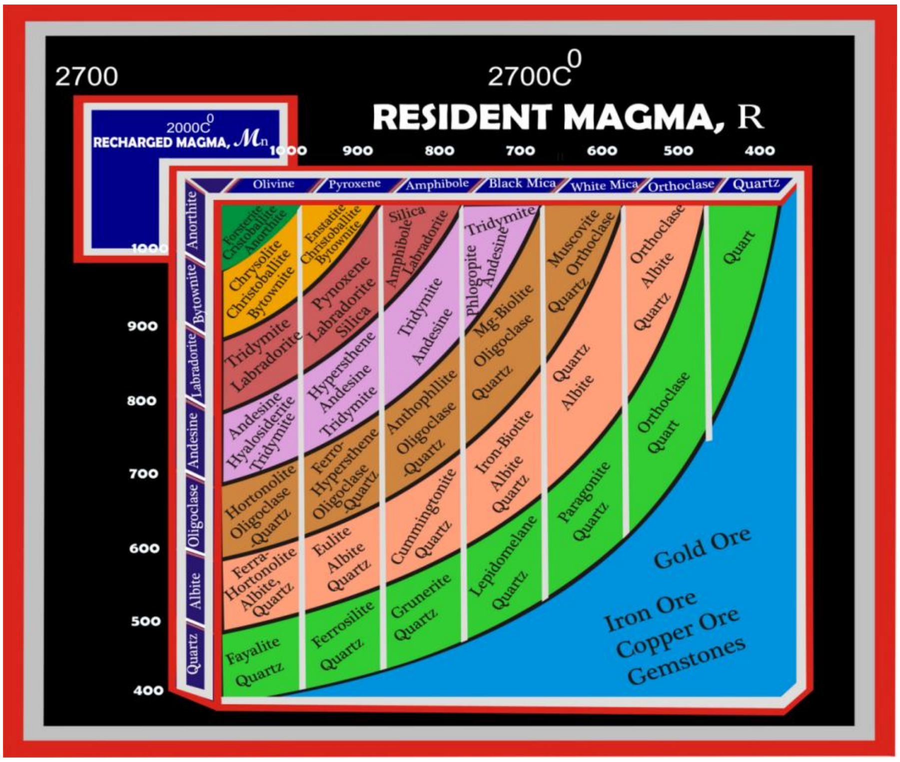


Figure 9. Model of Order of Crystallization and Mapping of Minerals under thermodynamic change

5. Conclution

In conclusion, findings have shown that both Goldschmidt and Bowen's concepts concerning distribution of elements and elemental substitution in igneous rocks can be mathematically connected throughout the formation of igneous rocks from the beginning to the end of crystallization through the following mathematical foundation under thermodynamic change.

1. Mathematical equations, Metrical matrix and Set notation methods find its application in geology especially the formation of rock forming minerals during crystallization of magma as they are applicable to other science and engineering disciplines.

2. Cauchy's definition of sequence can be applied during minerals formation from magma.

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